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Real-valued measurable cardinals and well-orderings of the reals. (English summary)

Set theory, 83–120, *Trends Math.*, *Birkhäuser*, *Basel*, 2006.

Summary: "We show that the existence of atomlessly measurable cardinals is incompatible with the existence of well-orderings of the reals in $L(\mathbb{R})$, but consistent with the existence of well-orderings of the reals that are third-order definable in the language of arithmetic. Specifically, we provide a general argument that, starting from a measurable cardinal, produces a forcing extension where $\mathfrak{c}$ is real-valued measurable and there is a Δ^2_2-well-ordering of $\mathbb{R}$. A variation of this idea, due to Woodin, gives Σ^2_1-well-orderings when applied to $L[\mu]$ or, more generally, $\Sigma^2_1(\mathrm{Hom}_\infty)$ if applied to nice inner models, provided enough large cardinals exist in V. We announce a recent result of Woodin indicating how to transform this variation into a proof from large cardinals of the Ω-consistency of real-valued measurability of $\mathfrak{c}$ together with the existence of Σ^2_1-definable well-orderings of $\mathbb{R}$. It follows that if the Ω-conjecture is true, and large cardinals are granted, then this statement can always be forced.

"However, we introduce a strengthening of real-valued measurability (real-valued hugeness), show its consistency, and prove that it contradicts the existence of third-order definable well-orderings of $\mathbb{R}$."

{For the entire collection see MR2267421 (2007e:03005)}