MR2585906 (Review)   03F30 (03D20 03D55)
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Goodstein's function. (English, Spanish summaries)
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The author gives a new proof of the following unprovability result: Goodstein's function $\mathcal{G}$ cannot be shown to be total in Peano Arithmetic (PA). He presents "explicit" formulas for $\mathcal{G}$. So, on the positive side, $\mathcal{G}$ is, indeed, total. On the other side, "explicit" indicates that formulas for $\mathcal{G}(n)$ themselves depend on the argument n. As n grows, formulas take more and more functions f_α from the Löb-Wainer fast growing hierarchy, and so $\mathcal{G}$ grows faster than any f_α. Since S. S. Wainer showed [Arch. Math. Logik Grundlagenforsch. **13** (1970), 136–153; MR0294134 (45 #3207)] that the f_α's dominate functions that are provably total in PA, $\mathcal{G}$ cannot be provably total.

The unprovability result was first shown by Kirby and Paris using indicators, a model theoretic device, and then by Cichon by comparing the growth of $\mathcal{G}(n)$ with the Hardy hierarchy.

The paper is very well written, and reading it is fun. The author gives the definition of $\mathcal{G}(n)$ in detail, and also the first four values. ($\mathcal{G}(4)$ is already horrendously large.) The significance of unprovability and how to compute various functions, etc., are accounted for.

Reviewed by *M. Yasuhara*