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Some applications of Sargsyan's equiconsistency method. (English summary)
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The study of indestructibility started with [R. J. Laver, Israel J. Math. **29** (1978), no. 4, 385–388; MR0472529 (57 #12226)]. Apter and his collaborators have worked extensively on it. This paper is a contribution to this study.

Following Hamkins, I refer to the property of being a strong cardinal as "strongness" rather than "strength". The key large cardinals used in the paper are hyperstrong cardinals, introduced by Baldwin, and a generalization, the new notion of hypercompact cardinals. Recall from [S. L. Baldwin, J. Symbolic Logic **51** (1986), no. 3, 547–559; MR0853838 (87m:03072)] that a cardinal κ is 0-hyperstrong iff it is strong. It is α-hyperstrong (for $\alpha > 0$) iff for any $\delta > \kappa$, there is an elementary $j\colon V \to M$ witnessing that κ is δ-strong, and such that, in M, κ is β-hyperstrong for all $\beta < \alpha$. We say that κ is hyperstrong iff it is α-hyperstrong for all α.

The notions of α-hypercompactness (for α an ordinal) and hypercompactness are obtained by replacing "strong" with "supercompact" everywhere in this definition.

The main results of the paper are as follows: Theorem 2 establishes that if κ is hypercompact, GCH holds, and there are no measurable cardinals larger than κ, then there is a forcing extension by a poset of size κ, where κ remains a supercompact limit of supercompact cardinals, the classes of supercompact and strongly compact cardinals agree except at measurable limit points, every supercompact cardinal δ is indestructible under δ-directed closed forcing, and every non-supercompact strongly compact cardinal δ has both its strong compactness and its degree of supercompactness indestructible under δ-directed closed forcing.

Theorem 6 shows that the existence of hyperstrong cardinals is equiconsistent with the conjunction of the following: there is a strong limit of strong cardinals, every strong cardinal is weakly indestructible, and the degree of strongness of every measurable limit of strong cardinals is weakly indestructible. Here, the weak indestructibility of the degree of strongness of δ means that this does not decrease under forcing by $<\delta$-strategically closed, (δ,∞)-distributive posets.

Theorem 2 improves a similar result shown in [A. W. Apter, Arch. Math. Logic **41** (2002), no. 8, 705–719; MR1949333 (2003j:03068)], where the conclusion was reached from the (significantly) stronger assumption that κ is almost huge. Both theorems use in essential ways methods of Sargsyan introduced in [A. W. Apter and G. Sargsyan, J. Symbolic Logic **75** (2010), no. 1, 314–322; MR2605896 (2011g:03129)]. Theorem 6 also uses inner model theory.

(On page 211, in the paragraph following the definition of $\mathbb{P}_{\gamma,\delta}$, note that the + in "$\gamma+\delta$" is really an "and".) *Andrés Eduardo Caicedo*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.