MR2944490 (Review) 03E35 00A30 03E45 03E65
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Foundational implications of the inner model hypothesis. (English summary)
Ann. Pure Appl. Logic **163** (2012), *no. 10,* 1360–1366.

This rather brief paper is a contribution to the philosophy of set theory. The brisk presentation seems to me to be a weakness rather than an asset, as many subtleties go unexplored.

The paper recalls Friedman's Inner Model Hypothesis (IMH) first introduced in [S.-D. Friedman, Bull. Symbolic Logic **12** (2006), no. 4, 591–600; MR2283091 (2007j:03065)], and contrasts it to two common views on the existence of independent statements from the standard axiomatization of set theory.

One of these views, advocated in particular by Shelah, suggests that "set theoretic universe" is an inherently undetermined notion, beyond what ZFC can prove. The other view espouses Gödel's program of extending ZFC by adopting large cardinal axioms, and expects that suitable further extensions will be possible.

The paper presents a weak criticism of the common approach to large cardinals. For example, it is stated that about the only evidence for their "correctness" is their success in providing us with a rich mathematical theory. Indeed, if this were the only argument in their favor, it would be a rather weak one, and advocating a highly restrictive axiom such as, say, $V = L$, would be similar. For a more nuanced presentation of Gödel's program and its success, it is better to refer to "Mathematics needs new axioms", J. R. Steel's contribution to [S. Feferman et al., Bull. Symbolic Logic **6** (2000), no. 4, 401–446; MR1814122 (2002a:03007)].

The paper indicates that "maximize", one of the intuitive principles ("rules of thumb") discussed, for example, in [P. J. Maddy, J. Symbolic Logic **53** (1988), no. 2, 481–511; MR0947855 (89i:03007); J. Symbolic Logic **53** (1988), no. 3, 736–764; MR0960996 (89m:03007)], and usually advanced to defend the adoption of large cardinal axioms, can also be used to justify statements (such as IMH) incompatible with such axioms. The IMH itself is briefly discussed (without mathematical details) in section 5 of the paper. It states that if any parameter-free sentence φ holds in an inner model of an outer model of the universe, then it already holds in an inner model. This is a maximality principle, stating that V is maximal with respect to *internal consistency*.

As the paper states, whether IMH is eventually accepted as a feature of the set theoretic universe will depend in large part on its *mathematical success*. Whether this occurs, Friedman's recent work will play a key role in this development. See for example [S.-D. Friedman and K. Thompson, J. Symbolic Logic **73** (2008), no. 3, 831–844; MR2444271 (2010e:03062)]. *Andrés Eduardo Caicedo*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.