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Smullyan, Raymond [[Smullyan, Raymond Merrill](#)] (1-IN-NDM)

★**Reflections—the magic, music and mathematics of Raymond Smullyan.**

World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2015. x+213 pp.
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In a charming and somewhat rambling style, the author recounts his life, paying particular attention to his three great passions: mathematics, magic, and music. There are also many, many puns.

Smullyan was born in 1919 in Far Rockaway, New York, and was 95 at the time the book was written. It is remarkable how lucid he is and how sharp his memory remains. “My earliest recollection is of being in my crib and being disturbed by the color of the wallpaper!” With many detours to tell jokes, pose logic puzzles, recall some technical details of the proofs of Kurt Gödel’s first incompleteness theorem, and so on, the book tells of Smullyan’s childhood, his education, his work in logic and as a professional magician, his passion for music and his involvement with the Piano Society (<http://pianosociety.com>).

The book consists of ten chapters. The first two are devoted to the author’s childhood. Smullyan traces his passion for music to his parents: “Both of my parents were musical—my father played the violin and my mother the piano.” As a child he also showed interest in science (particularly chemistry) and electronics (he even built a radio, from used parts). Already in these two chapters it becomes clear that the narrative of the book is anything but linear and that Smullyan does not limit himself to recounting his life. For instance, in Chapter 1 he uses an April Fool’s joke his brother played on him as an excuse to discuss the incompleteness theorem (a more technical discussion takes place in Chapter 5).

Chapter 3 is devoted to high school, which Smullyan did not finish. “It was partially my interest in Galois Theory and other branches of mathematics that caused me to drop out of high school in my junior year and to study on my own. I never did graduate from high school, but got into college by taking College Board Exams.” A recurring theme of the early chapters of the book is that Smullyan is a self-described “born-again dropout” all throughout his early education, including college, which he also never finished. He explains in Chapter 3 that during his self-study period after leaving high-school but before applying for college, he rediscovered Boolean rings. He also invented several chess puzzles, which were collected together years later in two books. As is the case with just about everything he describes, there is an interesting story here, regarding their publication. It is told later in Chapter 6.

Chapter 4 is about his college years. “The first college I attended was a small one called Pacific University. . . . I stayed at Pacific University for one semester and then transferred over to Reed College I remained at Reed for less than a semester.” While at Reed, Smullyan met the pianist Bernhard Abramowitch, who offered him a scholarship. This resulted in Smullyan moving to San Francisco, and then to Berkeley, to audit courses at the university. In particular, he audited a course on mathematical logic and as a result rediscovered Emil Post’s theorem on the completeness of propositional logic. While at Berkeley, his roommate was Leon Kirchner, who would later become a famous composer. After being in the Bay Area for a year, Smullyan moved back to New York, where he remained for a while (while sometimes attending lectures at the University of Chicago). He then returned to college, this time at Wisconsin. (The fact

that he took some philosophy courses at Wisconsin is then used as an excuse to present many puns involving logicians and philosophers. For instance: “What famous logician never needed to bathe?... Steven Cole Cleaney.”)

Chapter 5, “University of Chicago, Music, Magic”, opens with Smullyan telling us that after three semesters he transferred from Wisconsin to Chicago “and a year or two later I went back to New York”. Of particular note is that while at Chicago he took three philosophy courses from Rudolph Carnap. For the mathematically minded reader, this and the next chapter are the most interesting. Smullyan includes a letter sent to David Edmonds, of philosophybites.com, recounting his memories of Carnap and including recollections of his own first serious research on logic:

“[T]he three courses I took with Carnap were enormously beneficial! The first course was in mathematical logic, the second was in syntax, and the third was a seminar in syntax and semantics. . . . For the first course I wrote a paper on a new decision procedure for monadic first-order logic, which yielded, as corollaries, two decision procedures of Quine. Professor Carnap . . . wrote that I should send a copy of my paper to Quine, which I did. . . .

“[F]or the next two courses, I failed to hand in papers at the end of the course, and he rightly gave me *incompletes* in both courses. Several months after the end of the last course, I visited Carnap in Princeton and told him, ‘I have now completed two excellent papers, which I will soon send you.’ [Yes, I was sufficiently immodest to classify my papers as ‘excellent’!]. . . he returned them with A’s on both, and evidently remembered my calling my papers ‘excellent’, because he wrote me, ‘Your self-evaluation was correct.’ He added . . . ‘I think you should consider publishing this in the *Journal of Symbolic Logic*.’ On the other he wrote, ‘Again, I strongly urge you to publish this!’

“Well, I did publish the first one in the *Journal of Symbolic Logic* under the title ‘Languages in which Self-Reference is Possible [J. Symb. Logic **22** (1957), 55–67; [MR0090561](#)],’ and it was very well received. As for the second, I published parts of it in papers and parts in my book *Theory of Formal Systems* [Annals of Mathematics Studies, No. 47, Princeton Univ. Press, Princeton, NJ, 1961; [MR0121300](#); revised edition, Princeton Univ. Press, Princeton, NJ, 1961; [MR0152429](#); see also *Theory of formal systems*, Ph.D. thesis, Princeton Univ., 1959; [MR2612897](#)].

“Professor Carnap has been helpful to me in so many ways! When he was in Princeton, he showed my term paper ‘Languages in which Self-Reference is Possible’ to Gödel. Some years later, when I was a graduate student at Princeton, I met Gödel, who congratulated me on my paper, which he thought was my Ph.D. thesis!”

Also of note is that, on Carnap’s recommendation, John Kemeny at Dartmouth College invited him for an interview, which resulted in Smullyan teaching at Dartmouth for two years. The University of Chicago eventually granted him a bachelor’s degree, based partly on the courses taught at Dartmouth. Carnap was also responsible for introducing him to the incompleteness theorems. At this point in the narrative Smullyan proceeds to discuss these results, first using his well-known device of islands of knights and knaves, then through a discussion of a recursively enumerable set of true sentences, and finally through some puzzles involving coins.

Let me say a few words about the second approach, which should also strongly remind the reader of Gödel’s diagonal lemma. In turn, diagonalization plays a key role in the technical discussions in Chapter 6 and a very different role in Chapter 8. Consider a language consisting solely of 4 symbols: P, N, R, and * (there is a small typo on page 51 where an F appears instead of a P), and consider also a machine that prints words in this language. We introduce a simple semantics, where P stands for printable (our counterpart in this example for “provable”), R stands for repeat (thus R*X would mean the repeat of the word X, that is, XX), and N stands for not. A sentence is any

word of one of the following forms: $P*X$, $NP*X$, $PR*X$, and $NPR*X$, where X is an arbitrary word. These stand respectively for “ X is printable”, “ X is not printable”, “the repeat of X is printable”, and “the repeat of X is not printable”. The symbol $*$ is used for punctuation, to remove ambiguities (consider $P*RX$, claiming that the word “ RX ” is printable, versus $PR*X$, claiming that the word “ XX ” is printable). Printable is interpreted with respect to the given machine, that is, the machine prints (outputs) words in the language, and a word X is printable iff X is eventually one of the words the machine prints. The only requirement we impose on the machine is that any sentence it happens to print must be true. (Note that the machine may perhaps print many words that are not sentences.)

This simple semantics allows one to assign a truth value to each sentence. For instance, if $*$ is ever printed, then $P**$ is true, while if PP is never printed, then $P*PP$ and $PR*P$ are false, which in turns tells us that, for example, $NP*P*PP$ is true, etc. What is remarkable is that this system offers us an example of incompleteness: Although every printable sentence is true, there are true sentences that are not printable. An example Smullyan gives is the sentence $NPR*NPR*$, which is true if and only if the repeat of the word $NPR*$, that is, the word $NPR*NPR*$ itself, is not printable. The point is that, since the machine is sound, the sentence must be true. More elaborate examples are possible. The reader may want to come up with their own variation of Smullyan’s suggestion of exhibiting “two sentences X and Y such that one of the two is true but not printable, but there is no way of knowing which it is!”

Chapter 5 also discusses Smullyan’s professional foray into magic and meeting his first wife, and includes descriptions of several notable magic tricks.

Chapter 6 discusses Smullyan’s life at Darmouth and Princeton, where he went to graduate school: “At Princeton University I studied with two of the foremost logicians—Alonzo Church and Steve Kleene. Officially, Church was my thesis advisor, but most of my work was done during the year that Church was away. The mathematician who had the most influence on me, though, was the brilliant logician John Myhill. . . .” The chapter includes a good discussion of the diagonal lemma (leaving out the technicalities of Gödel numberings), which he uses to sketch a proof of Alfred Tarski’s theorem on undefinability of truth. There are several anecdotes regarding Gödel, some of them well known. It was Kemeny who introduced Gödel and Albert Einstein. “Einstein and Gödel had offices opposite each other in a hall of The Institute for Advanced Study. At the time Kemeny was there, Gödel was working on some strange physical theories about universes which were non-existent, but logically possible. One day Kemeny asked Gödel what Einstein thought of the work. Gödel replied that he didn’t know, since he had never met Einstein.”

The chapter also describes the beginning of Smullyan’s prolific logic puzzles book-writing career, starting with *What is the name of this book?* [Prentice Hall, Englewood Cliffs, NJ, 1978], which Smullyan uses as an excuse to present several more of his witty knights and knaves puzzles.

Chapter 7 is a relatively brief overview of Smullyan’s professional years, complete with several musical anecdotes.

Chapter 8 is devoted to the Diagonalization Society, a group Smullyan started with his former student Bruce Horowitz. The term “diagonalization” is borrowed from logic and applied to his own style of mostly verbal pranks. What is meant by it is “a sort of composite of *trick*, *fool*, *confuse*, *confound*, *mystify*, *get the better of*, and other things”. There then follows a disarming description of Smullyan’s courtship of and life with the love of his life, his second wife Blanche, a musician, “who passed away in January 2006 at the ripe old age of 100! We were married for 48 years.” The chapter closes with some brief recollections of W. Hoskisson, great grandson of Brigham Young, the Mormon

leader; this is perhaps the only detour of the book through Smullyan's other passion, the philosophy of religion.

The longest two chapters of the book are also its final ones. Chapter 9 is devoted to the Piano Society, and of course also includes many puns and puzzles. Chapter 10 is about "Lovely Ladies I Have Known". "I have been an incorrigible flirt all my life. I happen to be a born-again romantic. I can't help it. It's genetic!" Ostensibly on the subject of flirting, the chapter also includes many anecdotes from some of his former students and quotations from their letters. It is a nice way to round out and conclude a charming book.

I first encountered Smullyan's books while still in high school, and it wasn't until years later than I found out he was a professional logician, and much later still that I learned of his interest in magic and music. I enjoyed the book with his many detours and distractions, was reminded of the many hours I spent with *The lady or the tiger [and other logic puzzles]*, Knopf, New York, 1982], and even found a new exposition of the first incompleteness theorem. It was like meeting an old friend for the first time.

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