MR2777751 (2012i:03146) 03E60 (03C20 03E25)
Caicedo, Andrés Eduardo (1-BOISE); **Ketchersid, Richard** (1-MMOH-DM)
A trichotomy theorem in natural models of AD^+. (English summary)
Set theory and its applications, 227–258, *Contemp. Math.*, 533, *Amer. Math. Soc., Providence, RI*, 2011.

Cardinality arguments assuming the axiom of determinacy AD are quite different from their ZFC counterparts. During the last three decades, the influence of AD on the structure of the *cardinalities of well-orderable sets* (i.e. the *cardinals*) has been extensively investigated, but relatively little is known regarding the *cardinality of arbitrary sets*. This nicely written paper presents a few results that bear on these matters. In particular: if X is any uncountable set, then $\mathbb{R}$ or ω_1 can be embedded into X. This result is derived from a general trichotomy result for quasi-orders, which generalizes previous results of J. H. Silver [Ann. Math. Logic **18** (1980), no. 1, 1–28; MR0568914 (81d:03051)], M. D. Foreman [in *Logic, methodology and philosophy of science, VIII (Moscow, 1987)*, 223–244, Stud. Logic Found. Math., 126, North-Holland, Amsterdam, 1989; MR1034565 (90m:03086)] and G. Hjorth [J. Symbolic Logic **60** (1995), no. 4, 1199–1207; MR1367205 (97c:03126)]: given a quasi-order $(X, \leq)$, either it can be decomposed into a well-ordered union of quasi-chains, or it admits a perfect set of $\leq$-incomparable elements, or $\mathbb{R}/E_0$ embeds into X, where E_0 is the Vitali equivalence relation. Using these techniques the authors prove that the countable-finite games [M. Scheepers, J. Symbolic Logic **56** (1991), no. 3, 786–794; MR1129143 (92m:03075)] are not determined. All these results are proved under AD^+ (a technical strengthening of AD due to Woodin) with the additional assumption that either $\mathrm{V} = \mathrm{L}(\mathcal{P}(\mathbb{R}))$ or else $\mathrm{V} = \mathrm{L}(T, \mathbb{R})$, with T a set of ordinals. The authors should be commended for providing a highly readable introduction to the theory of AD^+.

{For the entire collection see MR2777741 (2012c:03006)}

Reviewed by *Alessandro Andretta*