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Constructing (ω_1, β)-morasses for $\omega_1 \leq \beta$. (English summary)
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In previous work [Far East J. Math. Sci. (FJMS) **54** (2011), no. 2, 117–148; MR2827553] (hereafter [PM]), B. Irrgang defined the notion of (ω_1, β)-morasses for $\omega_1 \leq \beta$. This extends R. B. Jensen's definition [see "Higher-gap morasses", handwritten notes, 1972/73; per bibl.], which only considers the notion for $\beta < \omega_1$. The reasons for this limitation, and how to circumvent them, are explained in [PM].

In [PM], the notion of κ-standard morasses is also introduced. It is shown there that any $\omega_{1+\beta}$-standard morass is an (ω_1, β)-morass, and that the existence of κ-standard morasses implies the existence of sets X such that $L_\kappa[X]$ computes cardinals correctly, and admits fine structure and condensation. See [PM] for details on these notions. Their definition is recalled in Section 1 of the paper under review.

The main result of this paper is that, conversely, if κ is a cardinal and $L_\kappa[X]$ satisfies these three properties, then there is a κ-standard morass (in $L_\kappa[X]$; see Section 3). In particular, the notion of (ω_1, β)-morasses is consistent for all $\beta \geq \omega_1$, and they exist in L.

Fine structure is reviewed in Section 2, and described in detail for $L[X]$ (applications, such as the existence of $\square$-sequences, are presented in Irrgang's dissertation [*Kondensation und Moräste*, Ph.D. dissertation, Ludwig-Maximilians-Univ. München, 2002]). The definitions of morasses and standard morasses are reviewed in Section 3. The construction is elaborate but resembles the definition of $\square$-sequences in L. The paper is carefully written and well organized. *Andrés Eduardo Caicedo*