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Caicedo, Andrés Eduardo (1-BOISE); **Clemens, John Daniel** (D-MUNS-ML); **Conley, Clinton Taylor** (1-CRNL-NDM); **Miller, Benjamin David** (D-MUNS-ML)
Definability of small puncture sets. (English summary)
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We say that a set P punctures a family of sets $\mathcal{A}$ if its intersection with each set in $\mathcal{A}$ is non-empty. Let κ be an infinite cardinal, and let $\mathfrak{X}$ be a class of Hausdorff spaces. A pointclass Γ is called κ-chromatic-on-$\mathfrak{X}$ if every Γ-measurable $\aleph_0$-dimensional digraph on a space $X \in \mathfrak{X}$ satisfies a weak analogue of the Kechris-Solecki-Todorcevic dichotomy theorem [A. S. Kechris, S. Solecki and S. B. Todorčević, Adv. Math. **141** (1999), no. 1, 1–44; MR1667145 (2000e:03132)] characterizing the existence of Borel colorings relative to κ. Let $\check{\Gamma}$ denote the pointclass of complements of sets in Γ. We say that $Y \subseteq X$ is weakly $\aleph_0$-universally Baire if $\pi^{-1}(Y)$ has the Baire property for every continuous function $\pi\colon 2^{\mathbb{N}} \to X$.

The following general theorem is proved.

Suppose that Γ is a κ-chromatic-on-$\mathfrak{X}$ pointclass, $X \in \mathfrak{X}$, E is a $\check{\Gamma}$-measurable, weakly $\aleph$-universally Baire equivalence relation on X, and $\mathcal{A} \subseteq [X/E]^{\leq \aleph_0}$ is Γ-measurable. Then at least one of the following holds:

(1) There is a set of cardinality strictly less than κ puncturing $\mathcal{A}$.

(2) There is a pairwise disjoint subset of $\mathcal{A}$ of cardinality $\mathfrak{c}$.

Moreover, if there is no surjection from a cardinal strictly less than κ to $\mathfrak{c}$, then exactly one of the above conditions holds.

This result is then used to establish refinements for several distinguished pointclasses. Also, the authors discuss the definability of the puncture sets obtained from the proofs.

Marek Balcerzak

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.