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★ **Foundations of mathematics.**

Essays in honor of W. Hugh Woodin's 60th birthday.

Proceedings of the Logic at Harvard Conference held at Harvard University, Cambridge, MA, March 27–29, 2015.

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From the preface: “This volume presents a collection of papers related to the work of W. Hugh Woodin, who has been one of the leading figures in set theory since the early 1980s. Woodin’s earliest work, when he was still an undergraduate at Caltech, was on automatic continuity in Banach spaces. It resulted in his earliest (unpublished) paper, which won the Eric Temple Bell Undergraduate Mathematics Research Prize in 1976 (and is mentioned in Garth Dales’s contribution to this volume). Later, but still early in his career, he studied possible behaviors for the continuum function, showing with Matthew Foreman that consistently (relative to the existence of suitable large cardinals) the Generalized Continuum Hypothesis can fail at every infinite cardinal; their work in this area and early insights by Woodin on Radin forcing led to the development of Radin- and Prikry-like forcings, and also provided some of the key insights that resulted in Moti Gitik’s theorems on the precise consistency strength of the Singular Cardinal Hypothesis. With Ted Slaman he has studied degree structures in recursion theory, with a focus on what are now called the biinterpretability conjectures.

“His central work, however, has been in the theory of large cardinals and determinacy. Among his many results in these areas, the most striking might be his identification of the existence of infinitely many Woodin cardinals as the exact consistency strength of the Axiom of Determinacy. The proof of this theorem involves two bodies of work which Woodin developed at essentially the same time. For the upper bound, Woodin developed the general notion of stationary tower embeddings, building on work of Foreman, Menachem Magidor and Saharon Shelah. For the lower bound, Woodin developed an analysis of the inner model HOD in determinacy models, building on work of Robert Solovay, Howard Becker and others. Among Woodin’s other important contributions to set theory are  $AD^+$ , arguably the right version of the axiom of determinacy and central to modern developments in the area; the theory of  $\mathbb{P}_{\max}$  forcing, a method for forcing over models of determinacy to produce models of the Axiom of Choice which are maximal for the structure  $\mathcal{P}(\omega_1)$ ; the extender algebra, which can be used to show that for certain models of set theory, every set is generic over an elementary extension; the theory of universally Baire sets, first defined in joint work with Qi Feng and Magidor and which has proved to be essential in the study of generic absoluteness; and the core-model induction, a method for deriving large-cardinal consistency strength from combinatorial principles. Each of these contributions has given rise to entire research areas.

“In recent years Woodin has been advancing a philosophical program that makes the case for new axioms that settle statements at the level of the Continuum Hypothesis and far beyond. With his work on the search for an ultimate inner model for large cardinals (the so-called Ultimate- $L$  model) as well as with the associated  $V = \text{Ultimate-}L$

Conjecture, Woodin is in fact making a case for new axioms that would arguably resolve (with the help of large cardinal axioms) all statements left undecided by the standard Zermelo-Fraenkel axioms with the Axiom of Choice (ZFC). This program also includes a careful analysis of extender models for very large cardinals.

“A meeting to celebrate Hugh’s 60th birthday was held during March 27–29, 2015, in the Fong Auditorium at Harvard University. The meeting featured nine one-hour lectures on topics reflecting the various stages of Hugh’s career. The speakers included some of his closest collaborators (Dales, Foreman, Alekos Kechris, Magidor, Tony Martin, Slaman) and others (Ronald Jensen, Grigor Sargsyan, John Steel) who have worked in inner model theory and determinacy, in areas close to the central concerns in Hugh’s research throughout his career. Feng, who has written two important papers with Hugh, was scheduled to speak but could not attend. The meeting was attended by Hugh’s advisor Solovay, several of his students (Joan Bagaria, Scott Cramer, Vincenzo Dimonte, Joel David Hamkins, George Kafkoulis, Paul Larson, Xianghui Shi, Liuzhen Wu) and a large number of people working in areas related to Woodin’s work.”

**Papers in this collection include the following:**

H. G. Dales, “Norming infinitesimals of large fields”, 1–29. [MR3656305](#)

Theodore A. Slaman and Mariya I. Soskova, “The enumeration degrees: local and global structural interactions”, 31–67. [MR3656306](#)

A. S. Kechris, M. Sokić and S. Todorcevic, “Ramsey properties of finite measure algebras and topological dynamics of the group of measure preserving automorphisms: some results and an open problem”, 69–85. [MR3656307](#)

Andrés Eduardo Caicedo and Jacob Hilton, “Topological Ramsey numbers and countable ordinals”, 87–120. [MR3656308](#)

Victoria Gitman and Joel David Hamkins, “Open determinacy for class games”, 121–143. [MR3656309](#)

M. Malliaris and S. Shelah, “Open problems on ultrafilters and some connections to the continuum”, 145–159. [MR3656310](#)

P. D. Welch, “Obtaining Woodin’s cardinals”, 161–176. [MR3656311](#)

Ralf Schindler, “Woodin’s axiom (\*), or Martin’s maximum, or both?”, 177–204. [MR3656312](#)

Grigor Sargsyan, “Translation procedures in descriptive inner model theory”, 205–223. [MR3656313](#)

Scott Cramer, “Implications of very large cardinals”, 225–257. [MR3656314](#)

Justin Tatch Moore, “What makes the continuum  $\aleph_2$ ”, 259–287. [MR3656315](#)

Penelope Maddy, “Set-theoretic foundations”, 289–322. [MR3656316](#)

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