Caicedo, Andrés Eduardo; **Larson, Paul**; **Sargsyan, Grigor**; **Schindler, Ralf**; **Steel, John**; **Zeman, Martin**

Square principles in $\mathbb{P}_{\max}$ extensions. (English) Zbl 06709006

Isr. J. Math. 217, 231-261 (2017).

In [The axiom of determinacy, forcing axioms, and the nonstationary ideal. 2nd revised ed. Berlin: Walter de Gruyter (2010; Zbl 1203.03059)], *W. H. Woodin* applied his forcing notion $\mathbb{P}_{\max}$ to a model of $\mathrm{AD}_{\mathbb{R}}$ + "Θ is regular″ to obtain an extension where $\mathrm{MM}^{++}(\mathfrak{c})$, holds. Here $\mathrm{AD}_{\mathbb{R}}$ is the Axiom of Determinacy for sets of reals, Θ is the least ordinal that is not a surjective image of $\mathbb{R}$, $\mathfrak{c}$ is the cardinality of $\mathbb{R}$, and $\mathrm{MM}^{++}(\mathfrak{c})$ is the assertion that $(H(\omega_2), \in, NSI)$ is Σ_1 elementary in $(H(\omega_2), \in, NSI)$ of V^P for any stationary set preserving P of size at most $\mathfrak{c}$. (NSI is the nonstationary ideal on ω_1.)

In this article, the authors apply $\mathbb{P}_{\max}$ to theories stronger than $\mathrm{AD}_{\mathbb{R}}$ + "Θ is regular" to obtain some consequences of $\mathrm{MM}^{++}(\mathfrak{c}^+)$, namely results about Jensen's square principles. In particular, they produce a model of $2^{\aleph_0} = 2^{\aleph_1} = \aleph_2 + \neg\Box(\omega_2) + \neg\Box(\omega_3)$.

Reviewer: J. M. Plotkin (East Lansing)

MSC:

03E35 Consistency; independence results (set theory)
03E60 Axiom of determinacy, etc.
03E45 Constructibility, ordinal definability, and related notions
03E55 Large cardinals

Keywords:

axiom of determinacy; $\mathbb{P}_{\max}$; square principles

Full Text: DOI arXiv

References:

[1] J. E. Baumgartner, A new class of order types, Ann.Math. Logic 9 (1976), 187–222. · Zbl 0339.04002· doi:10.1016/0003-4843(76)90001-2

[2] A. E. Caicedo and R. Ketchersid, A trichotomy theorem in natural models of AD+, in Set Theory and its Applications (L. Babinkostova, A. E. Caicedo, S. Geschke and M. Scheepers, eds.), Contemp. Math., Vol. 533, Amer. Math. Soc., Providence, RI, 2011, pp. 227–258. · Zbl 1245.03079

[3] J. Cummings, Compactness and incompactness phenomena in set theory, in Logic Colloquium' 01 (M. Baaz, S.-D. Friedman and J. Kraj\'{\i}\v{c}ek, eds.), Lect. Notes Log., Vol. 20, Assoc. Symbol. Logic, Urbana, IL, 2005, pp. 139–150. · Zbl 1081.03046

[4] S. Jackson, Structural consequences of AD, in Handbook of Set Theory. Vols. 1, 2, 3 (M. Foreman and A. Kanamori, eds.), Springer, Dordrecht, 2010, pp. 1753–1876. · Zbl 1198.03056

[5] T. Jech, Set Theory, The third millennium edition, revised and expanded, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2003.

[6] R. B. Jensen, The fine structure of the constructible hierarchy, with a section by Jack Silver, Ann. Math. Logic 4 (1972), 229–308; erratum, ibid. 4 (1972), 443. · Zbl 0257.02035· doi:10.1016/0003-4843(72)90001-0

[7] R. B. Jensen, Corrections and remarks, unpublished manuscript, 1998, available at http://www.mathematik.hu-berlin.de/$\sim$raesch/org/jensen.html.

[8] A. Kanamori, The Higher Infinite, Large cardinals in set theory from their beginnings, second ed. paperback reprint of the 2003 edition, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2009. · Zbl 1154.03033

[9] A. S. Kechris, Classical Descriptive Set Theory, Graduate Texts in Mathematics, Vol. 156, Springer-Verlag, New York, 1995. · Zbl 0819.04002

[10] R. Ketchersid, More structural consequences of AD, in Set Theory and its Applications (L. Babinkostova, A. E. Caicedo, S. Geschke and M. Scheepers, eds.), Contemp. Math., Vol. 533, Amer. Math. Soc., Providence, RI, 2011, pp. 71–105. · Zbl 1244.03135

[11] P. Koellner and W. H. Woodin, Large cardinals from determinacy, in Handbook of Set Theory. Vols. 1, 2, 3 (M.

Foreman and A. Kanamori, eds.), Springer, Dordrecht, 2010, pp. 1951–2119. · Zbl 1198.03072

[12] B. K\"onig and Y. Yoshinobu, Fragments of Martin's Maximum in generic extensions, MLQ Math. Log. Q. 50 (2004), 297–302. · Zbl 1047.03039· doi:10.1002/malq.200410101

[13] P. Larson, Separating stationary reflection principles, J. Symbolic Logic 65 (2000), 247–258. · Zbl 0945.03076· doi:10.2307/2586534

[14] P. B. Larson, Forcing over models of determinacy, in Handbook of Set Theory. Vols. 1, 2, 3 (M. Foreman and A. Kanamori, eds.), Springer, Dordrecht, 2010, pp. 2121–2177. · Zbl 1198.03063

[15] Y. N. Moschovakis, Descriptive Set Theory, second ed., Mathematical Surveys and Monographs, Vol. 155, American Mathematical Society, Providence, RI, 2009. · Zbl 1172.03026

[16] W. J. Mitchell and J. R. Steel, Fine Structure and Iteration Trees, Lecture Notes in Logic, Vol. 3, Springer-Verlag, Berlin, 1994. · Zbl 0805.03042

[17] G. Sargsyan, Comparison theory of hod mice below AD+ + "the largest Suslin cardinal is a member of the Solovay sequence", available at http://math.rutgers.edu/$\sim$gs481/comparison.pdf, 2015.

[18] G. Sargsyan, Hod mice and the Mouse Set Conjecture, Mem. Amer.Math. Soc. 236 (2015), viii+172. · Zbl 06479624

[19] G. Sargsyan, The analysis of HOD below the theory AD+ + "the largest Suslin cardinal is a member of the Solovay sequence", available at http://math.rutgers.edu/$\sim$gs481/HOD.pdf, 2015.

[20] E. Schimmerling, Coherent sequences and threads, Adv. Math. 216 (2007), 89–117. · Zbl 1124.03022· doi:10.1016/j.aim.2007.05.005

[21] R. M. Solovay, The independence of DC from AD, in Cabal Seminar 76–77 (Proc. Caltech-UCLA Logic Sem., 1976–77) (A. S. Kechris and Y. N. Moschovakis, eds.), Lecture Notes in Math., Vol. 689, Springer, Berlin, 1978, pp. 171–183. · Zbl 0388.03022

[22] R. Schindler and J. R. Steel, The Core Model Induction, unpublished book, 2014, available at http://wwwmath.uni-muenster.de/u/rds/.

[23] G. Sargsyan and J. R. Steel, The mouse set conjecture for sets of reals, J. Symb. Log. 80 (2015), 671–683. · Zbl 06472571· doi:10.1017/jsl.2014.78

[24] G. Sargsyan and N. Trang, The largest Suslin axiom, book in preparation. · Zbl 1325.03057

[25] J. R. Steel, The Core Model Iterability Problem, Lecture Notes in Logic, Vol. 8, Springer-Verlag, Berlin, 1996. · Zbl 0864.03035

[26] J. R. Steel, An outline of inner model theory, in Handbook of Set Theory. Vols. 1, 2, 3 (M. Foreman and A. Kanamori, eds.), Springer, Dordrecht, 2010, pp. 1595–1684. · Zbl 1198.03070

[27] E. Schimmerling and M. Zeman, C · Zbl 1095.03050· doi:10.1142/S0219061304000310

[28] S. Todor\v{c}evi\'c, A note on the Proper Forcing Axiom, in Axiomatic Set Theory (Boulder, Colo., 1983) (J. E. Baumgartner, D. A. Martin and S. Shelah, eds.), Contemp. Math., Vol. 31, Amer. Math. Soc., Providence, RI, 1984, pp. 209–218.

[29] S. Todor\v{c}evi\'c, Localized reflection and fragments of PFA, in Set Theory (Piscataway, NJ, 1999) (S. Thomas, ed.), DIMACS Ser. Discrete Math. Theoret. Comput. Sci., Vol. 58, Amer. Math. Soc., Providence, RI, 2002, pp. 135–148.

[30] W. H. Woodin, The Axiom of Determinacy, Forcing Axioms, and the Nonstationary Ideal, second revised ed., de Gruyter Series in Logic and its Applications, Vol. 1, Walter de Gruyter GmbH \& Co. KG, Berlin, 2010.

[31] M. Zeman, Inner Models and Large Cardinals, de Gruyter Series in Logic and its Applications, Vol. 5, Walter de Gruyter \& Co., Berlin, 2002.

[32] M. Zeman, Global square sequences in extender models, Ann. Pure Appl. Logic 161 (2010), 956–985. · Zbl 1263.03031· doi:10.1016/j.apal.2009.12.003