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**The bounded proper forcing axiom and well orderings of the reals.** (English) Zbl 1113.03039  
Math. Res. Lett. 13, No. 2-3, 393-408 (2006).

Summary: We show that the bounded proper forcing axiom BPFA implies that there is a well-ordering of  $\mathcal{P}(\omega_1)$  which is  $\Delta_1$ -definable with parameter a subset of  $\omega_1$ . Our proof shows that if BPFA holds then any inner model of the universe of sets that correctly computes  $\aleph_2$  and also satisfies BPFA must contain all subsets of  $\omega_1$ . We show as applications how to build minimal models of BPFA and that BPFA implies that the decision problem for the H\"artig quantifier is not lightface projective.

**MSC:**

- 03E05 Combinatorial set theory (logic)
- 03E45 Constructibility, ordinal definability, and related notions
- 03E47 Other notions of set-theoretic definability
- 03E65 Other hypotheses and axioms (set theory)
- 03E40 Other aspects of forcing (set theory)
- 03C80 Logic with extra quantifiers and operators

Cited in **1** Review  
Cited in **3** Documents

**Keywords:**

MRP; definable well-orderings; BPFA; inner model; minimal models; decision problem; H\"artig quantifier

**Full Text:** [DOI](#)