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Regressive functions on pairs. (English) Zbl 1247.05255

Eur. J. Comb. 31, No. 3, 803-812 (2010).

Let $[l,m]=\{l,l+1,\dots,m\}$ and $X^{[k]}$ be the collection of k-sized subsets of X. A function $f:X^{[k]}\to N$ is called regressive if $f(s)<\min(s)$ whenever $s\in X^{[k]}$ and $\min(s)>0$. For such an f, a subset $H\subseteq X$ is called min-homogeneous for f if $0\notin H$ and, whenever $s,t\in H^{[k]}$ and $\min(s)=\min(t)$, then $f(s)=f(t)$. The two-variable function $g(n,m)$ is defined as the least l such that any regressive function $f:[m,l]^{[2]}\to[0,l-2]$ admits a min-homogeneous set of size n and $G(n,m)$ is the least l such that for any regressive $f:[m,l]^{[2]}\to[0,l-2]$, there is a min-homogeneous set for f of size n whose minimum element is m (in the paper it is shown that this function is well defined). In the literature, the values of g (more precisely, the values of $g(\cdot,2)$) are referred to as "regressive Ramsey numbers". In this paper it is shown by combinatorial arguments, that $g(4,3)=37$, $g(4,4)\le 85$, $g(5,2)\le 41\times 2^{37}-1$, $G(4,m)=2^m(m+2)-1$ and for all n, there is a constant c_n such that $G(n,m)<A_{n-1}(c_nm)$ for almost all m. Here, $A_n=A(n,\cdot)$, where A is Ackermann's function. Also, $g(n,m)\ge A_{n-1}(m-1)$ holds for all $n\ge 2$. These results also establish the rate of growth of the function $g(n,\cdot)$ as being precisely that of the $(n-1)$st level of the Ackermann hierarchy of fast growing functions.

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MSC:

05D10 Ramsey theory

Keywords:

regressive function; regressive Ramsey number; Ackermann's function; primitive recursive function; Ackermannian growth; min-homogeneous set

Full Text: DOI

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