

Caicedo, Andrés Eduardo; Friedman, Sy-David BPFA and projective well-orderings of the reals. (English) Zbl 1252.03107 J. Symb. Log. 76, No. 4, 1126-1136 (2011).

Summary: If the bounded proper forcing axiom BPFA holds and  $\omega_1 = \omega_1^L$ , then there is a lightface  $\Sigma_3^1$  well-ordering of the reals. The argument combines a well-ordering due to Caicedo-Veličković with an absoluteness result for models of MA in the spirit of "David's trick". We also present a general coding scheme that allows us to show that BPFA is equiconsistent with R being lightface  $\Sigma_4^1$ , for many "consistently locally certified" relations R on  $\mathbb{R}$ . This is accomplished through a use of David's trick and a coding through the  $\Sigma_2$ -stable ordinals of L.

## MSC:

03E15 Descriptive set theory (logic)

Cited in 1 Document

03E50 Continuum hypothesis; Martin's axiom (logic)

03E57 Generic absoluteness; forcing axioms

## **Keywords:**

projective well-orderings of the reals; bounded proper forcing axiom

Full Text: DOI