The Bulletin of Symbolic Logic
Volume 12, Number 2, June 2006

REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the Journal in 1936 until the end of 1999. The Association moved the reviews to this Bulletin, beginning in 2000.

 Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the Journal, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this Bulletin, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the Journal, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the Journal, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the Journal, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the Journal, vol. 1, pp. 121–218).

John Nolt. ***Logics.*** Wadsworth, 1997, xii + 468 pp.

Classical logic has been the standard fare for introductory courses in logic at universities now for over 50 years. But nonclassical logics have come to be of interest to mathematicians and philosophers for a foundational comparison to classical logic and for their applications. It now seems intellectually dishonest to present classical logic as the settled sole formalization of reasoning in an introductory course on logic. This book is meant to be used to give a broader introduction to logic.

Logics is valuable throughout in connecting logic to reasoning. Nolt begins with good discussions of arguments, validity, and much of the background necessary to the study of formal logic. He then turns to a presentation of classical logic up through the completeness theorem for predicate logic.

Nolt's semantics for predicate logic are new. Rather than interpreting variables as in the Tarskian approach, or varying over different interpretations of names as in Mates's approach or Shoenfield's approach, he expands the valuations, that is, the interpretation of the predicates and names, by considering potential names, evaluating formulas by using such potential names for objects. But no fixed supply of such names is given and the formal language is never expanded to include them, so that the potential names are in a limbo between syntax and semantics. This flaw makes his semantics unintelligible.

For his inference systems he uses natural deduction. He devotes much space to extremely long syntactic analyses, both for examples and for completeness theorems. Oddly, he doesn't prove a deduction theorem or a strong completeness theorem for any of the logics he considers.

1079-8986/06/1202-0006/$3.00

After classical logic Nolt turns to extensions of classical logic: modal logics, deontic and tense logics, and second-order logic. Then he presents nonclassical logics: free logics, multi-valued logics, supervaluations, infinite-valued and fuzzy logics, intuitionistic logics, relevance logics, and nonmonotonic logics.

Nolt's motivation of the various logics and their use in analyses of philosophical issues is often good and in the case of the modal logics (including deontic and tense logics) of considerable interest even to those long familiar with the systems. However, in the discussion of modal systems he does not consider the principal criticism of them as involving use-mention confusions, which can be highlighted with an analysis of deduction theorems for them. In the discussion of higher-order logics he misses the main philosophical issue of why we should identify properties and relations with sets and what we mean by all subsets and all predicates.

For the semantics of most of these logics Nolt introduces the predicate interpretation without separately considering the propositional analysis. This is hard to follow, particularly when there are many controversial choices to be made for the quantifiers that obscure the analysis of the propositional connectives. Only for some of the logics that he classifies as nonclassical does he consider just the propositional logic, and those presentations are more accessible.

It is not clear what audience Nolt has in mind for this text. The density of the material and the depth of the philosophical discussions suggest that this book would be suitable only for upper-division undergraduates in philosophy. The complexity of the predicate semantics for modal logics in particular seems at a level accessible only for advanced students of logic.

Overall, this text does not appear to be suitable for a first introduction to logic. But the motivations of many of the logics make it a useful reference for faculty, whether preparing such a course or engaging in research.

RICHARD L. EPSTEIN

Advanced Reasoning Forum, P. O. Box 635, Socorro, NM 87801, USA.
rle@AdvancedReasoningForum.org.

BOB HALE AND CRISPIN WRIGHT. ***The reason's proper study: Essays toward a neo-Fregean philosophy of mathematics.*** Oxford University Press, New York. 2001, 472 pp.

For most of his career Frege believed that (i) arithmetic is a body of truths concerned with independently existing objects and that (ii) arithmetic is derivable from logical principles and definitions alone. This is what Frege meant by the claim that arithmetic is analytic. Moreover, he offered an original defense of this claim. First he argued that ascriptions of number, which for Frege are ascriptions of a number to a concept, are governed by Hume's principle:

> HUME'S PRINCIPLE: The number of F = the number of G if and only if F and G are in one-to-one correspondence.

For all its merits, Hume's principle seems unable to settle mixed identity questions of the form "The number of $F = q$", where q is a singular term purported to refer to an object of an ostensibly different sort. In the face of what has come to be known as the Julius Caesar problem, Frege sought to provide an explicit definition of numerical terms of the form "the number of F":

> The number of F is the extension of the concept *being equinumerous with F*.

This definition takes place against the background of Frege's theory of extensions which would eventually emerge as second-order logic supplemented with Basic Law V:

> BASIC LAW V: The extension of F = the extension of G if and only if F and$G$ are coextensive.

Frege's explicit definition of number had Hume's principle as an immediate consequence. In ***Grundlagen***, Frege outlined a derivation of ordinary arithmetic from his definition of number. Unfortunately, by the time the two volumes of Frege's ***Grundgesetze*** appeared in print with all the technical details of the derivation, Russell had shown Basic Law V to be inconsistent. Frege tried to modify Basic Law V to avoid the contradiction but to no avail. Faced with Russell's discovery, he later abandoned his life's project to ground arithmetic on logical principles and definitions.

Bob Hale and Crispin Wright have led a recent effort to revive Frege's ambition to establish the analyticity of arithmetic. Neo-Fregeanism, as this effort is now known, is a renewed defense of Frege's two main tenets, but with a qualified understanding of the analyticity of arithmetic. Where Hale and Wright differ from Frege is in their assessment of the prospects for such a defense. Frege's mistake, they suggest, had been to seek an explicit definition of number in the face of the Julius Caesar problem. Instead, they propose to make do with Hume's principle alone as a definitional basis for arithmetic. Two technical observations provide some grounds for optimism. The first observation is that Hume's principle is consistent if second-order arithmetic is. Since the consistency of this theory seems beyond serious question, this observation has been taken to show the consistency of Hume's principle itself. The second observation is that, in the context of second-order logic, Hume's principle does indeed suffice for a derivation of all the Dedekind-Peano axioms. This is what, following a suggestion of George Boolos, has come to be known as *Frege's Theorem*.

The question remains whether Frege's Theorem may be reasonably taken to underwrite a vindication of the analyticity of arithmetic. The present volume collects together 15 papers by Bob Hale and Crispin Wright which, at the time of publication, represented their best effort in that direction. That effort had taken place in a span of several years and all but one of the papers in the collection had appeared in print before the publication of this volume. The book includes an introduction and a postcript written especially for the occasion. The introduction surveys the targets of the neo-Fregean program and explains how the different papers in the volume contribute to the fulfillment of its philosophical obligations. The postcript, in contrast, provides a useful summary of 18 problems still facing the neo-Fregean program and sketches directions for further research. The topics addressed in the papers reflect the breadth of ontological and epistemological questions raised by the neo-Fregean philosophy of arithmetic. The papers are divided into five main parts.

Part I (Ontology and Abstraction Principles) consists of five papers largely centered on two challenges. One has to do with the neo-Fregean conception of object as just what a singular term refers to. The central challenge is to be able to identify expressions as singular terms independently of the assumption that they refer or purport to refer to objects. Another challenge for the neo-Fregean is related to Frege's remark in ***Grundlagen*** §64 that the two sides of instances of abstraction principles in general and Hume's principle in particular *carve* a single content in two different ways. A quick glance at an instance of Hume's principle suggests that the left and right hand side of the biconditional should diverge in their truth conditions in view of the fact that one side is a statement of numerical identity while the other is a statement of one-one correspondence. The challenge is to motivate an understanding of *truth condition* on which the two sides of an instance of Hume's principle may reasonably be said to have identical truth conditions.

Part II (Responses to Critics) consists of four papers written mostly in response to criticisms by Hartry Field and Michael Dummett. One theme in common with the first part is Hartry's Field objection based in the apparent divergence in truth conditions between the two sides of instances of Hume's principle. Field had urged the rejection of that principle in favor of a conditionalized version of the form: "If numbers exist, then the number of $F =$ the number of G if and only if they are in one-to-one correspondence." Other concerns are more general and not specific to the neo-Fregean program. One of the papers by the first author discusses Field's generalization of Benacerraf's classic epistemological challenge for platonism. Two more papers react to Michael Dummett's criticisms of Frege's philosophy of arithmetic and the more recent neo-Fregean revival. Some of Dummett's objections center on the viability of a neo-Fregean vindication of Frege's platonism and have to do with what he thinks is the inability of Hume's principle to secure a robust reference for the numerical terms or to counter ontological reductionism. A related objection concerns the viability of a neo-Fregean solution of the Julius Caesar problem. Discussion of what is probably Dummett's most serious objection is postponed to Part III, where Wright considers the question of whether the neo-Fregean project is marred by the impredicativity of Hume's principle.

Part III (On Hume's Principle) consists of four papers written by Crispin Wright on Hume's principle. This part is largely centered on two themes. One is the impredicativity of abstraction of Hume's principle, which Michael Dummett has found objectionable as a sort of vicious circularity that threatens the prospects of both Frege's philosophy of arithmetic and its neo-Fregean revival. Wright argues for the legitimacy of impredicative abstraction principles in general and denies that impredicativity places a serious obstacle for the neo-Fregean defense of the analyticity of arithmetic. The other theme of this part has to do with a critical obligation for the neo-Fregean, who must provide a reasonable understanding of analyticity on which Hume's principle may appropriately be called analytic. The neo-Fregean answer is that Hume's principle serves as an implicit definition of the concept of number. The task of course is to elaborate and defend the neo-Fregean account of *implicit definition* on which Hume's principle indeed qualifies as an implicit definition.

Part IV (On the Differentiation of Abstracts) consists of a single joint effort to survey what they seem to the authors the best lines of response to the Julius Caesar problem, which is what led Frege to seek an explicit definition of number in the first place. If Hume's principle is to succeed as a complete explanation of the concept of number, then one might expect it to settle the truth conditions of mixed identity statements of the form "the number of $F = q$", where "q" is a term explicitly purported to denote an object of an ostensibly different sort. Hale and Wright hope that the criterion of identity associated with the sortal concept *number* as introduced by Hume's principle imposes some restrictions on the extension of the concept of number that suffice, for example, to exclude the possibility that a number be identical with an object that falls under a sortal concept governed by a different criterion of identity.

Part V (Beyond Number Theory) consists of an attempt by Bob Hale to extend the neo-Fregean program beyond arithmetic to cover real analysis. The neo-Fregean ambition is to identify further abstraction principles with a claim to provide a foundation for other branches of ordinary mathematics and perhaps even set theory. In the specific case of analysis, Hale identifies and studies an abstraction principle which generates real numbers as ratios of quantities.

A few years have now elapsed since the publication of Hale and Wright's volume and much work has been done to address some of the problems they raise in their postcript. The problems listed in the author's postcript are centered on three main fronts: (a) Abstraction principles and their credentials to serve as a foundation for ordinary mathematics, (b) the legitimacy of higher-order logic, and (c) the prospects for an extension of the neo-Fregean program for the rest of mathematics. Two problems of the first sort still strike this reviewer as particularly challenging. One is the question of what justifies the special epistemological status of philosophically virtuous abstraction principles as opposed to others. What confers on Hume's principle, for example, a privileged epistemological and ontological status as a foundation of arithmetic? Not only do we seem to lack an explanation of why some abstraction principles enjoy a more privileged epistemological status than others, we seem to have little reason to expect a principled and informative distinction between philosophically virtuous abstraction principles and all of the rest. Michael Dummett once raised the objection that next to what are supposed to be philosophically virtuous abstraction principles like

Hume's principle lie inconsistent principles like Basic Law V. Unfortunately, a series of refinements of Dummett's *bad company objection* would seem to suggest that there is little hope to set philosophical virtuous abstraction principles apart from less virtuous principles.

More progress has been achieved on some of the other fronts. Hale's approach to real analysis has been recently supplemented by an alternative neo-Fregean account of the real numbers based on Dedekind's construction of the real numbers as Dedekind cuts. Crispin Wright and Stewart Shapiro, for example, have explored this route in which successive abstraction principles eventually take one from cardinals to Dedekind cuts which are then identified with real numbers. The prospects for a neo-Fregean account of set theory are a different matter entirely. Very briefly, the challenge is to identify an acceptable abstraction principle that might serve as a basis for a well-motivated neo-Fregean set theory from which to recover a sufficient amount of set theory to deserve the name.

GABRIEL UZQUIANO

Department of Philosophy, Ohio State University, 230 N. Oval Mall, Columbus, OH 43210, USA. uzquiano-cruz.1@osu.edu.

GRAHAM PRIEST. ***An introduction to non-classical logic.*** Cambridge University Press, 2001, xxi + 242 pp.

The book offers a systematic presentation of non-classical *propositional* logics ("classical" logic understood just as the standard propositional logic as commonly taught). For most of the logics presented, the possible world semantics is used and the proof systems are based on tableaux. Each chapter dealing with a logic ends with a section with proofs of theorems, a historical survey and suggestions for further reading. The notion of a formula is standard, with one remarkable feature that the author never uses the name "implication" and instead calls the connective "conditional". In each chapter there is a shorter or longer discussion on how the connective of conditional corresponds (and in fact does not correspond) to the intuitive notion of the conditional "if – then" of the natural language, understood as relating some proposition (the consequent) to some other proposition (the antecedent) on which, in some sense, it depends (p. 9). This seems to be a reasonable philosophical question (as a part of logical analysis of natural language), but one can ask if this is a *logical* question. The author explicitly says (p. 1) that "the point of logic is to give an account of the notion of validity: what follows from what". But from this point of view, implication is, in each reasonable logic, a well defined connective and modus ponens just expresses its behavior w.r.t. validity. The *paradoxes* of implication just show that implication does not formalize the intuitive notion of conditional, which is interesting (more or less) but is not a fault.

The chapters are as follows: *Chapter* 1 – *Classical logic and material conditional*. Formulas, trees and tableau rules for classical logic are defined; the discussion on paradoxes starts. *Chapter* 2 – *Basic modal logic*. Possible world semantics is defined and the system K (after "Kripke") is introduced (no restriction to the accessibility relation). Philosophical discussion on the meaning of possible worlds—three approaches discussed (realism, actualism, meinongianism). And of course tableau rules, completeness proof, as in all chapters (not stressed below). *Chapter* 3 – *Normal modal logics*. These are K and stronger logics putting some restrictions to the accessibility relation, in particular the famous S5 is presented. There is a discussion on the notion of necessity. *Chapter* 4 – *Non-normal worlds, strict conditionals*. Each structure of possible worlds contains a subset of "normal" possible worlds; in a non-normal world, each formula is possible and no formula is necessary. Lewis's S2 and S3 are introduced and Lewis's notion of strict conditional (defined as $\Box(A \supset B)$) is discussed. *Chapter* 5 – *Conditional logics*. There is a new connective $>$ and the possible world structure has a system $\{R_A \mid A \text{ a formula}\}$ of relations, i.e., each formula has its accessibility relation. $A > B$ is true in a world w if B is true in each world R_A-accessible from w. *Chapter* 6 – *Intuitionistic logic*. Usual possible world semantics for this logic presented (R reflexive and transitive, evaluation of formulas in possible worlds is hereditary: if p is true in w then it is true in each world accessible from w). The intuitionist negation is in fact $\Box\neg A$ and the intuitionist implication (conditional) is in fact the strict conditional. Even here the author discusses paradoxes of the conditional. *Chapter* 7 – *Many-valued logic*. After a general explanation of what many valued logics are only the three-valued logics by Kleene and by Łukasiewicz are presented and discussed, as well the variant of Kleene logic with the two-element set of designated values (values different from 0). There is an extensive discussion on the third value and its meaning as "neither true nor false" or, alternatively, as "both true and false" (and the corresponding paradoxes). *Chapter* 8 – *First degree entailment*. This is apparently a rather unknown logic, with two truth values but the evaluation of propositional variables is not a function assigning to each variable a truth value but a relation between variables and truth values (thus a variable may be related to no value, to the value 0, the value 1 or to two values—four possibilities). The relation extends to formulas built by conjunction, disjunction and negation using natural rules. This logic is "the core of a family of relevant logics" (p. 159) and is representable as a four-valued logic. *Chapters* 9 *and* 10 deal with relevant logics. In Chapter 9, the logic of Chapter 8 is extended by a variant of the strict conditional and in Chapter 10 there is a ternary relation used to formulate truth conditions for $\rightarrow$. We omit any details.

Chapter 11 – *Fuzzy logic*. Here we shall be more detailed and critical. The author understands (correctly) fuzzy logic as a logic motivated by vagueness (Sorites paradox). He presents the infinitely valued Łukasiewicz logic and its axioms; its completeness (with respect to 1 as the designated value) is only stated, with reference to literature. Then he suggests that for any $\varepsilon > 0$, the set $\{x \mid \varepsilon \leq x \leq 1\}$ can be taken as the set of designated values but then modus ponens fails. He finds this counter-intuitive. But it is not; if one makes consequences from not fully true assumptions, the degree of truth may decrease. This was well elaborated already in the late seventies in the pioneering series of papers by Pavelka (*Pavelka J.*: On Fuzzy Logic I, II, III, Zeitschrift für Mathematische Logik und Grundlagen der Mathematik 25 (1979), 45–52, 119–134, 447–464) who was one of first authors relating fuzzy logic to residuated lattices. The weighted modus ponens says that if A is at least r-true and $A \supset B$ is at least s-true then B is at least $r * s$-true where $*$ is so-called Łukasiewicz t-norm $x * y = \max(0, x + y - 1)$ (the truth function of strong Łukasiewicz conjunction) whose residuum is Łukasiewicz implication. One can consider evaluated proofs defined in the obvious way and get a variant of completeness. Moreover: The monograph *Hájek P.*: Metamathematics of fuzzy logic (Kluwer 1998) became the beginning of intensive development of t-norm based fuzzy logics. Łukasiewicz logic is one of them, but there are others, notably Gödel fuzzy logic, which is a logic satisfying modus ponens for Priest's sets of designated truth values mentioned above. These logics have double semantics, the standard semantics with the real unit interval as the set of truth degrees and the general semantics given by so-called BL-algebras (residuated lattices satisfying some additional axioms). It is a pity that Priest's book (which appeared in 2001 and possibly was finished earlier) does not mention this development and thus is, with respect to fuzzy logic, not very informative.

At the end of the preface the author says: "If one waited for perfection one would wait forever". His book is, in spite of the criticism just presented, a very valuable source in many directions. The reviewer met recently Professor Priest and told him the criticism; he has replied that a new edition of the book is in preparation and the chapter on fuzzy logic will be modified accordingly. Then several works should be taken into consideration; among them, besides those mentioned above, the following two: *Gottwald S.*: Treatise on Many-Valued Logics. Studies in Logic and Computation vol. 9, Research Studies Press 2001; *Novák V, Perfilieva I., Močkoř J.*: Mathematical Principles of Fuzzy Logic. Kluwer 2000.

Petr Hájek

Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod vodárenskou věží 2, 182 07 Prague, Czech Republic. hajek@cs.cas.cz.

Fairouz Kamareddine, Twan Laan, and Rob Nederpelt. ***A modern perspective on type theory—From its origins until today.*** Applied Logic Series, vol. 29. Kluwer Academic Publishers, Dordrecht, 2004, xiv + 357 pp.

This book is a monograph on the historic role of type theory in the epistemological foundations of mathematics. As such it covers important aspects of type theory, but by no means all of type theory. For example, the role type theory plays in programming languages is not adressed. This is not meant as criticism of the book under review, but as mild criticism of its title. For a large part it is a consequence of the size of the book (xiv + 357 pp.). A more comprehensive treatment of type theory is to be expected from the forthcoming volume(s) edited by Barendregt, in total well over 1000 pages. There the historical and epistemological aspects will not be covered to the same extent as here, which makes the topic of the book under review particularly well-chosen.

The role which type theory has played and still plays in writing down mathematical knowledge in a consistent way can roughly be divided into two successive stages.

(1) Limiting the set of expressions, that is, propositions and their constituents, to the well-typed ones and thereby avoiding the paradoxes. For example, the Russell paradox in the form of the unrestricted comprehension schema $\exists y.\ \forall x.\ y(x) \Longleftrightarrow \phi$, leads to inconsistency by taking $\neg x(x)$ for ϕ. One way out is a type system in which expressions like $x(x)$ are not well-typed. This is the line of the Principia Mathematica by Russell and Whitehead, explained in a modern setting in Part I of the book.

(2) Adding an extra layer of expressions representing proofs. In (1), propositions *have* a type but do not play a role as types themselves. In constructive mathematics arose the insight that a proof of an implication $\phi \to \psi$ can be viewed as a function mapping proofs of ϕ to proofs of ψ. For example, the identity function on the set of proofs of ϕ can be viewed as a proof of $\phi \to \phi$. This gave rise to an interpretation called *Propositions as Types*, in which one views a proposition as the type of all its proofs. This interpretation does not only make sense for implication, but can be extended to all connectives and quantifiers. For example, the left projection function on pairs of proofs of ϕ and ψ can be viewed as a proof of $\phi \wedge \psi \to \phi$. This makes it possible to have one type system in which one can represent the domains of individuals, functions, propositions as well as the proofs. This is the line of the Automath system initiated by De Bruijn, based on the constructive interpretation of proofs by Kolmogorov and Heyting, formalized by Curry and Howard. This is what Part II of the book is about.

Part I.

Ch. 1: Gives a historical account of the relevant paradoxes.

Ch. 2: Presents in detail the type system of the Principia Mathematica and relates it to modern typed lambda calculus. This results in a rather complicated system, so-called Ramified Type Theory (RTT), which has both types and orders, but which is completely predicative.

Ch. 3: Deals with relaxing some of the constraints of the Principia. The relaxation is traditionally called *deramification* and boils down to leaving out the orders from the types. This leads to a type theory known as the Simple Theory of Types (STT), which is impredicative. For example, $\forall p : *.\ p$ is a proposition which quantifies over propositions. If we give this proposition the type $*$ then the range of the quantification includes the proposition itself. This is impredicative. In a predicative system one would give it a different type, one order higher, e.g., $(\forall p\colon * .\ p)\colon *'$. But then one may need also $(\forall p\colon *'.\ p) : *''$ and so on. Although impredicativity poses serious semantical challenges, it does not lead (at least not by itself) to inconsistencies. STT became later known as Church' Simple Theory of Types. The last section of this chapter reviews Kripke's Theory of Truth (KTT), in which one leaves out the types and retains the orders. Among other things it is shown that RTT can be embedded in KTT in a sound and complete way.

PART II. This part is based on the Propositions as Types (PAT) interpretation.

Ch. 4: Sketches the two variants of PAT: (i) identifying a proposition with the type of its proofs (Curry and Howard), and (ii) mapping a proposition ϕ to the type *proofs*(ϕ) of its proofs (De Bruijn). For the latter approach the phrase *Proofs as Terms* is more adequate. Typed lambda calculi of a special format, so-called Pure Type Systems (PTSs) and a classification of these, the Barendregt Cube, are introduced.

Ch. 5: Elaborates the PAT interpretation for STT and RTT. This involves the definition of a suitable typed lambda calculus λRTT and a formalization of the respective proof theories in this lambda calculus. The chapter culminates in the proof of the soundness of the embedding of RTT (and STT) in λRTT. The reviewer could not find a result on the completeness of the embedding, which presumably doesn't hold.

Ch. 6: Investigates the correspondence between RTT and the type system Nuprl based on Martin-Löfs type theory. The latter type theory was originally developed as a foundation of constructive mathematics and is one of the strongest predicative type theories. The orders are clearly visible in the universes typed $*_1 : *_2 : *_3 \ldots$. Again the chapter culminates in a soundness proof. Nuprl proves slightly (and harmlessy) more embedded judgements of RTT than RTT itself.

Ch. 7: Describes the system Automath initiated by De Bruijn. Automath is the first *proof assistant* (also called *logical framework*). The goal of the Automath project is to develop a universal language for mathematics, in such a way that proofs can be verified mechanically. This is the old ideal of Leibniz and Hilbert, where the ambition to decide mechanically on truth (which is known to be impossible in general) has been replaced by the mechanical verification of proofs (which is already difficult enough). De Bruijn discovered independently PAT with *proofs*(ϕ). Automath has been highly influential, also through its successors Coq and Nuprl. (Outside the scope of the book, but worth mentioning is that De Bruijn discovered a technique called *nameless dummies* which was the first correct implementation of bound variables.)

PART III. This part deals with extensions of Pure Type Systems with definitions (Ch. 8), parameters (Ch. 9) and the combination of the the two (Ch. 10).

REMARKS.

The overall presentation is very careful, systematic and detailed. This sometimes leads to a high degree of repetition, for example, the same set of PTS terms is defined on pages 112, 116, 311, 312, 328, and variations of this definition are abundant. It would have been possible to sacrifice some details and some variants of systems in favour of other important topics in type theory: inductive types, extensionality, the Logic Cube and Girard's Paradox. Martin Löf systems could be given a bigger role, for example as an introduction to the chapter on Nuprl. The methodological choice for PTSs has worked out very well for structuring the systems treated in the book. Unfortunately, as many of these systems are *almost* PTSs, many of the metaresults had to be verified again (this is no criticism, but an indication that the notion of PTS is not even general enough). I warmly recommend this book to anyone interested in the historic role of type theory in the foundations of mathematics.

MARC BEZEM

Institute of Informatics, University of Bergen, Postboks 7800, N-5020 Bergen, Norway. bezem@ii.uib.no.

Linear logic in computer science, edited by T. Ehrhard, J.-Y. Girard, P. Ruet, and P. Scott, London Mathematical Society Lecture Note Series, 316. Cambridge University Press, 2004, 392 pp.

18 years have passed since linear logic was introduced by Jean-Yves Girard in a 1987 ***Theoretical Computer Science*** article. When in 2005 (on its 30th anniversary) the journal

compiled the list of its 100 most cited articles, Girard's article turned out to top it. This book illustrates how far linear logic has penetrated computer science and how the relationship has been shaping up over the years. As such, it should appeal to anyone interested in applications of logic and, especially, the logical foundations of computer science.

The attractiveness of linear logic for computer science stems from the special emphasis the logic puts on the management of formulas. In short, the contraction and weakening rules are severely restricted and can only be applied to formulas of a special shape. Apart from that, a linear logic proof must use assumptions *linearly*, i.e., exactly once. This turns out to open up the path to a fine-grained analysis of intuitionistic and classical logics, and novel ways of reasoning about the structure of proofs. In particular, linear logic brings in a new graph-theoretic concept of a proof in the form of a *proof-net*. Proof-nets are designed to capture the "crucial" elements in a proof and to eliminate the problem of inessential differences arising in sequent calculus presentations (the so-called commuting equivalences). Their structure has helped to analyze cut-elimination in logics, which quickly translated into intuitions about reduction strategies for programs. An effective use of linear logic has been made in the field of mathematical semantics of programs, where it has helped systematize the knowledge and is now routinely used in descriptions of the semantic structure of models. Linear logic has also become a basis for the design of resource-sensitive type systems.

This book is a representative collection of research highlights inspired by linear logic. It begins with a series of tutorial papers, which should be especially useful to readers wishing to get a quick overview of the role that linear logic has played in several different contexts. The tutorial part is followed by some very recent contributions that demonstrate the current state of the art in understanding a number of topics that arose in research into linear logic. Finally, the closing invited articles present new areas of application in which taking advantage of linear logic offers a new perspective on the subject.

The first tutorial by Rick Blute and Phil Scott is a comprehensive survey of connections between linear logic and category theory. Quite remarkably, the proof theory of linear logic corresponds to many well-established notions in category theory, which creates a fertile ground for interaction between the two subjects. All of the category-theoretic concepts needed to analyze linear logic are introduced and discussed in this tutorial, as well as illustrated with many examples. It will provide the reader with indispensible vocabulary and methodology to track and understand the recent progress in investigations into the semantics of proofs and programming languages. The numerous category-theoretic links have by now become part of the folklore in the field and made it much easier for researchers to communicate about correspondences between type theories and semantic universes.

While the first tutorial concerns linear logic in the context of mathematical semantics, in the second one Stefano Guerrini describes in detail how to use linear logic to analyze the operational semantics of the λ-calculus. Two properties of linear logic make this application possible. Firstly, λ-terms can be readily translated into it, for example, via the $A \Rightarrow B = {!A} \multimap B$ translation. Secondly, thanks to the geometric nature of proof-nets, cut-elimination can be carried out by graph rewriting. The combination of the two turns out to reveal powerful geometric insights about λ-term reduction, which in particular can be used to identify optimal reduction strategies.

The first two tutorials present linear logic in the tradition of the 'computation as cut-elimination' paradigm, which constitutes a logical foundation for functional programming. The next one surveys the influence that the logic has had on logic programming, which regards proof search as the logical counterpart of computation. Here Dale Miller gives an overview of the development of linear logic programming languages and explains the new kinds of search strategies that the richer structure of linear logic allows for. Towards the end, he also lists the fields of application in which linear logic programming has made a difference.

In the final tutorial Glynn Winskel exposes the ubiquity of linearity in distributed computation, where it is unrealistic to expect processes to have unrestricted ability to copy other processes. He introduces a process language with built-in linearity restrictions and demonstrates that, in spite of them, it is expressive enough to represent many common idioms of distributed computation. The language is subsequently studied semantically using domain-theoretic techniques inspired by the semantics of linear logic.

Altogether, the tutorials can be recommended as points of entry to the subject, especially for readers with a logical background. A little more background knowledge will be needed to understand the contribution of the four articles following them, which offer an insight into the sort of problems that have been studied by the linear logic community.

In one of them, Paul-André Melliès revisits the issue of proof-net correctness and proposes a new topological correctness criterion based on homeomorphism. Proof-nets are also the subject of another article, by Olivier Laurent and Lorenzo Tortora de Falco, in which the authors set out to identify the right notion of a proof-net in polarized linear logic. The interest in the polarized fragment stems from the fact that classical logic can be translated into it and the fragment is easier to analyze than full propositional linear logic. The next article by Jean-Marc Andreoli introduces Coloured Linear Logics, which in turn are well-behaved extensions of linear logic satisfying the desirable properties of cut-elimination and focussing. The new logics are investigated through a novel sequent calculus based on explicit addresses of occurrences of formulas in proofs. This way of referencing is fundamental to Girard's programme of *ludics*, started in 2001, which aims to be a novel interactive and resource-sensitive account of logic that refines linear logic. Some aspects of ludics, the issue of uniformity in particular, are explained in this book in an article by Claudia Faggian et al., who develop instructive examples in this new area.

The book closes with two invited articles. Jim Lambek reports on the latest linguistic applications of deductive systems related to noncommutative linear logic and explains their polycategorical semantics, while Jean-Yves Girard introduces a new interpretation of linear logic inside the realm of quantum mechanics. The latter article illustrates one of the most recent research directions emerging from investigations into linear logic: attempts to recast quantum physics in terms of logic and category theory.

To summarize, the book successfully documents a very fruitful interplay of logic with computer science that has been brought about by the study of linear logic. One would like to see many more such examples in the future.

ANDRZEJ S. MURAWSKI

Oxford University Computing Laboratory, Wolfson Building, Parks Road, Oxford OX1 3QD, UK. Andrzej.Murawski@comlab.ox.ac.uk.

ITAY NEEMAN. ***The determinacy of long games.*** De Gruyter Series in Logic and its Applications, vol. 7. Walter de Gruyter, Berlin, 2004, xi + 317 pp.

The deep connection between large cardinals and determinacy has been a central concern of modern research in set theory. The simplest possible setting is easy to recall: A set $C \subset \omega^\omega$ is fixed, and two players I and II alternate choosing integers $x(n)$ for infinitely many stages, with player I playing first, thus defining a real $x = \langle x(n) \colon n \in \omega \rangle \in \omega^\omega$. We can call these infinitely many moves a *round*. Player I wins this round iff $x \in C$, otherwise player II wins. Let's denote this game by $G_\omega(C)$. A winning strategy for I is a function $\tau \colon \omega^{<\omega} \to \omega$ such that if at each stage n player I plays $x(2n) = \tau(\langle x(0), \dots, x(2n-1) \rangle)$, then the resulting real x is in C. Similarly we can define a winning strategy for player II, and $G_\omega(C)$ is *determined* iff one of the players has a winning strategy.

From a well-ordering of the reals it is easy (by a diagonal argument) to produce a non-determined set of reals. However, large cardinal axioms imply that all sets of reals in $L(\mathbb{R})$, and more, are determined. See, for example, Neeman's papers *Optimal proofs of determinacy*,

The Bulletin of Symbolic Logic, vol. 1 (1995), pp. 327–339 and *Optimal proofs of determinacy II*, ***The Journal of Mathematical Logic***, vol. 2 (2002), pp. 227–258 (from now on, OpdII). Conversely, from the determinacy of sufficiently closed pointclasses of reals, the existence of inner models with large cardinals can be established. See, for example, the forthcoming paper by Koellner and Woodin, *Large cardinals from determinacy*, to appear in the ***Handbook of Set Theory***, Kanamori, Foreman, Magidor, eds. All of this constitutes what one could call the *first generation* of results tying large cardinals and determinacy. This is of course informal notation, and not intended to convey the impression that nothing else can be said or done about first generation results.

The second generation of results, to which the book under review is a seminal contribution, probably begins with John Steel's paper *Long games*, in ***Cabal Seminar 81–85***, Kechris, Martin, Steel, eds., Springer (1988), pp. 56–97 (from now on, Lg). In the presence of large cardinal axioms, the first generation results focus on obtaining the determinacy of more and more complex real pointclasses, where the complexity is measured descriptive set theoretically. Here, the attention shifts from the complexity of the classes to that of the games themselves, by allowing them to run for longer than ω many stages.

This book is intended for a serious graduate student or a researcher in the area. It is very carefully written and although a serious effort has been made to motivate its arguments, they tend to be very long and technical, and a novice may feel lost at points. In *An introduction to proofs of determinacy of long games*, in ***Logic Colloquium '01***, Lecture Notes in Logic, vol. 20, Baaz, Friedman, Krajícek, eds., AK Peters (2005), pp. 43–88, Neeman has written a wonderful expository account of the theory described in this book, leaving out the technical details that comprise most of the book itself, and the reader is encouraged to look at this paper first, as a motivation for the work to come.

In terms of prerequisites for these technical details, the two papers by Martin and Steel JSL LVII 1332 and JSL LVII 1333, and Neeman's *Inner models in the region of a Woodin limit of Woodin cardinals*, ***Annals of Pure and Applied Logic***, vol. 116 (2002), pp. 67–155 (from now on, ImrWlWc) are essential, although if the reader is familiar with the basic theory of iteration trees and is willing to accept some results as black boxes, then Appendix A of the book will suffice.

The book begins with a lively historical introduction in which basic definitions are recalled. It explains how the central large cardinal concept of *Woodin cardinal* was isolated and why it is the key notion in determinacy results. In their pivotal papers cited above, Martin and Steel introduced the notion of *iteration tree*, which is also at the core of modern inner model theory. In short, nice inner models for large cardinal notions weaker than Woodin cardinals can be compared by iterating ultrapowers, very much as in Kunen's argument for models with measurable cardinals, see e.g., Jech, BSL XI 243. These are essentially *linear* iterations, and this linearity seriously bounds the complexity of the reals that can belong to such inner models (for example, in the fine structural context, all these reals are Δ^1_3 in a countable ordinal). Martin and Steel found a non-linear method of iterating ultrapowers of inner models with Woodin cardinals. These models give then rise to iteration trees, trees of structures with embeddings between the models appearing along their branches. If two models are compared this way, at limit stages of the comparison process, different possibilities on how to continue the trees may arise, and the existence of these choices increases the complexity of their comparison process and explains why these models allow more complicated reals than those appearing in linearly iterable models.

The comparison process between models with Woodin cardinals may be naturally described in terms of *iteration games* between two players, "good" and "bad". There are several kinds of iteration games, but essentially all consist of "bad" playing iteration trees and then "good" picking a branch leading to a well-founded model, which "bad" then uses as the root of a new tree to continue the game, with this going on for as many rounds as the game specifies. The game ends if an ill-founded model is produced, otherwise "good" wins. If "good" has a winning strategy, then the original model at the root of the first tree played by "bad" is *iterable* and the strategy is called an *iteration strategy*. Neeman's ImrWIWc shows that the existence of enough large cardinals implies the existence of *nice* models M, i.e., iterable models of enough set theory with as many Woodin cardinals as required for any of the applications in the book.

The key idea is now that these iteration strategies can be used to produce winning strategies for either I or II in *long* games of the kind one considers when interested in determinacy by means of what one could call a *translation procedure*. The simplest of these long games, $G_{\omega\cdot(n+1)}(C)$, for $C \subset (\omega^\omega)^{n+1}$, is defined similarly to $G_\omega(C)$, only that now $n+1$ rounds are played, one after the other, thus producing an $(n+1)$-tuple of reals $\langle x_0, \dots, x_n\rangle$. As before, player I wins iff this tuple is in C. Say that C is $\mathbf{\Pi}^1_1$. Then the determinacy of $G_{\omega\cdot(n+1)}(C)$ implies the determinacy of all the games $G_\omega(D)$ for D in $\mathbf{\Pi}^1_{n+1}$, since the quantifiers in the projective definition of D can be simulated with the runs that give rise to $x_1, \dots, x_n$.

The core of the book is a series of arguments showing how to translate iteration strategies for nice models M as above into winning strategies for either player in one of several possible kinds of long games. For the games $G_{\omega\cdot(n+1)}(C)$ just explained, this is described in detail in chapter 1, which also presents a concise introduction to the theory of Woodin cardinals. This argument strongly resembles the one in OpdII, and the reader may want to look at this paper as a warm-up. This chapter also introduces, in a simple setting, the notation that is required to understand subsequent chapters. I must say that at some points the author's choice of notation seemed to me slightly odd, but no non-standard usage occurs, and the presentation is detailed and careful enough that the reader may just as well ignore my comment.

Several games are introduced, they are each more elaborate than the ones from previous chapters. The games described in chapter 1 are thus designed to provide a proof of projective determinacy by showing the determinacy of games $G_{\omega\cdot(n+1)}(C)$ for C closed. In chapter 2 determinacy is shown for games of fixed countable length, and in chapter 3 this is generalized even further to games of continuously varied countable length. These correspond, and generalize, the long games of Steel's paper Lg. Finally, chapters 4 through 7 are devoted to the proof of determinacy of games whose length reaches a local cardinal, i.e., this length is uncountable in some inner model, but not necessarily in V. As a typical application, Exercise 7.F.14 shows the result of Woodin that (from the existence of an iterable M such that there is a countable θ that in M is Woodin and limit of Woodin cardinals), there is a proper class inner model P such that, in P, all definable games of length ω_1^P are determined. The techniques of the book have been further generalized in subsequent papers by Neeman, so this area of research is clearly very promising territory.

Along the way, the reader encounters a few additional topics of independent interest. For example, universally Baire sets and homogeneously Suslin sets are discussed in chapter 2, and in chapter 4 a very general presentation of Woodin's extender algebra (in many generators) is given. The only presentation of the extender algebra that I was aware of was what would be in this context the algebra in one generator. See, for example, Steel *An outline of inner model theory*, to appear in the ***Handbook of Set Theory***, Kanamori, Foreman, Magidor, eds.

Let me finally say a few words about the translation procedure. Throughout the book the setting consists of a nice inner model M as above, a set $\mathcal{W}$ of Woodin cardinals from M (there are some restrictions on the Woodin cardinals in $\mathcal{W}$, but we can ignore them here), some $\delta \in \mathcal{W}$ and some $\mathtt{col}(\omega, \delta)$-name $\dot{A}$ coding what is in essence a set of reals (associated to the set we have been calling C). Actually, $\dot{A}$ is a name for a subset of $(M \parallel \delta)^\omega \times \omega^\omega$, where $M \parallel \delta$ denotes V_δ^M. Essential to the arguments is the continuity of a series of approximating games $\mathcal{A}[s]$ for $s \in \omega^{<\omega}$. Continuity is important in that these games are given inside of M although they give rise to games $\mathcal{A}[x]$ for $x \in \omega^\omega$. This matters for two reasons. An obvious one is that sometimes $x \notin M$. A more subtle one is that M is replaced frequently by other models N coming from iteration trees on M, so the games $\mathcal{A}[s]$, being elements of M, can be moved into the corresponding new games on N via the same embeddings that move M. The games $\mathcal{A}[s]$ build on the argument from Martin and Steel, JSL LVII 332 and, as in Neeman's game in OpdII, either a winning strategy exists for player I in M for $\mathcal{A}[x]$ or a winning strategy exists for player II in an ultrapower of M for a game corresponding to the images of δ and of $\dot{A}$ under this ultrapower. The game $\mathcal{A}[x]$ consists of player I trying to witness that $x \in \dot{A}[h]$ for h generic over M with II trying to witness the opposite. Greatly simplifying what is actually done, in each stage, player I plays conditions in $\mathtt{col}(\omega, \delta)$ and a set of names for reals. These collections of names are decreasing, the conditions force that any name in the collection should be in $\dot{A}$, and that these names agree more and more with x. Player II plays dense sets and the conditions that player I plays should be contained in these dense sets, thus eventually producing a generic for $\mathtt{col}(\omega, \delta)$ as required if player I has a winning strategy.

The key notions that the book introduces are those of *pivot* and *mixing* games. The pivot games, intended in the case where player I has no winning strategy, build in addition an iteration tree on M (using extenders below δ), and a continuous witness that the relevant branch on which the outcome for player II is examined is well-founded. This is done using a key lemma from Martin and Steel, JSL LVII 332. Mixing games give player I some flexibility on how the iteration trees must be extended and in particular restrict the possible extenders to be used along the way. The pivot game is designed so that either player I has a winning strategy in $\mathcal{A}[x]$ or else player II ought to have a winning strategy in the *shifted* game. This is by no means automatic. For example, chapter 5 is devoted to what amounts to a proof that this is the case for the long games to be presented in chapter 7. What can ultimately be described as a correctness argument for M shows that this implies that either I or II has a winning strategy in the original game on C. The way these pivot and mixing games are combined is quite delicate. The extender algebra is used in some cases to produce reals that guide how these games are executed. The iterability of M is essential to even describe how the games on M corresponding to continuously played games are to be organized.

To close, let me add that this book is a very welcome addition to the literature in set theory and a magnificent example of the high standards that the de Gruyter Series is intending to achieve. It is by no means an easy book, but (as I mentioned above) it is very carefully written, its arguments are in essence self-contained, and motivation is presented, even though somewhat more sparingly as the book progresses; both the techniques and even the definitions are quite involved, but a patient and careful reader will surely be more than handsomely rewarded.

ANDRÉS EDUARDO CAICEDO

Department of Mathematics, Mail code 253-37, California Institute of Technology, Pasadena, CA 91125, USA. caicedo@caltech.edu.

DAVID J. PYM AND EIKE RITTER. ***Reductive logic and proof-search—Proof theory, semantics, and control.*** Oxford Logic Guides, vol. 45. Oxford Science Publications, 2004, 208 pp.

Since a long time mathematical logic has been formulated as a formalization of deductive reasoning: given hypotheses, how a conclusion is derived. But computational logic has emphasized the reductive reasoning: given a putative conclusion, what are sufficient premisses? In this monograph the authors aim at showing that the reductive view of logic is fundamental and also at providing a semantics of proof-search that is strongly related to the model theory of the studied logic. They focus on how to define such a semantics in intuitionistic logic which models the logical and operational aspects via an embedding of intuitionistic reductive logic into classical reductive logic. An illustrating example of this semantics of proof-search is given throughout a particular class of games. The presented work is a research monograph and even if it includes familiar, established ideas and material, it provides many research contributions on the subject of reductive logic and proof-search. As mentioned by L. Wallen in the foreword, the reader will have a motivated and significant introduction to new approaches to semantic investigations of syntactic formulations of logical systems to support efficient proof-search in formal logics.

In Chapter 1, the authors introduce their perspectives on reductive logic with respect to deductive logic and proof-search. Theorem proving, or algorithmic proof-search, is considered as a central technology within computational sciences and many problems, like parsing, type-checking, specification and correctness, can be formulated as judgements about formal texts, for instance represented in logical formalisms. In this context the authors emphasize that many interactive theorem provers and reasoning tools, that implement partial or total procedures to prove such judgements, have common logical bases, namely *reductive inference* and *reductive proof*. Moreover, calculi often formulated as deductive systems (for instance resolution) can be reformulated as reductive systems. A main goal of the authors is then to provide clear and usable model-theoretic foundations, and then related proof-theoretic solutions, to deal with the *partiality* of objects (reductive proofs) built during a reduction and also the key computational feature, namely the *control*, which leads from reductive logic to proof-search. The proposals to solve the related problems are firstly a framework based on a notion of Kripke world which can maintain lost information and secondly, for intuitionistic logic, an embedding of intuitionistic reductions inside classical reductions in which the control of intuitionistic search (mainly backtracking) is studied. The originality of the approach lies in the choice of the $\lambda\mu\nu$-calculus and its categorical models, both to relate intuitionistic and classical logic consequences and to take account of the backtracking semantics. Moreover specific games models are developed to provide a complementary perspective towards what the semantics of proof-search is. In order to present existing material and contributions, the authors also discuss some mathematical prerequisites for the readers: logical prerequisites about semantics and proof theory but also algebraic prerequisites about lattices, categories and functors.

In Chapter 2, the authors introduce the natural deduction proof theory of intuitionistic logic, with graphical and sequential systems, and the simply-typed λ-calculus that is a language of realizers for intuitionistic consequences. Starting from the idea of free deduction introduced by M. Parigot, they study classical logic in a similar way and then propose a language, the $\lambda\mu\nu$-calculus, of realizers for classical natural deduction. A key point is the use of $\lambda\mu$-calculus for classical logical and its non-trivial extension with the disjunction.

In Chapter 3, they focus on semantics for the systems introduced in Chapter 2 and define categorical models of the related calculi. They also propose games models for both intuitionistic and classical proofs. A new form of games semantics is defined combining features of Lorenzen's games for intuitionistic provability, Hyland and Ong's games for linear logic and Ong's games for $\lambda\mu$-calculus. It forms the basis of an example developed throughout the monograph. Then semantics of classical proofs is presented through new models of the $\lambda\mu$-calculus.

In Chapter 4, the authors present the main concepts about reductive proof by introducing Gentzen sequent calculus that appears significant as a basis for reductive proof. Moreover, a representation of sequent calculus proofs in the $\lambda\mu\nu$-calculus with explicit substitution is given. Then, they provide a systematic account of reductive proof theory and a systematic analysis of the analytic view of resolution by the notion of uniform proof and in particular a reconstruction of Mints intuitionistic resolution. This chapter ends with a discussion of the computational complexity of the proposed methods. It gives a systematic account of reductive proof theory.

In Chapter 5, they define a class of categorical models for reduction and then provide a systematic model-theoretic account of reductive logic. The main problem is to find semantic structures that take into account not only the space of proofs but also the space of reductions.

They consider specific structures for modelling reductions which generalize structures giving models of proofs and then defining models for both intuitionistic and classical reduction. An appropriate soundness theorem for reductions and also a completeness theorem via a term model construction are proved.

In Chapter 6, the authors analyze the addition of algorithmic control regimes to reductive proof then leading to proof-search. They present a semantics for proof-search in reductive logic which integrates backtracking within the model theory and thus captures the slogan "proof-search is reduction+control". Moreover, proof-search in intuitionistic logic is studied through embedding of intuitionistic reduction inside classical reduction and a game semantics for proof-search is provided.

There is no chapter about conclusions and perspectives but in a discussion in Chapter 1, the authors, after having summarized the main contributions, identify a few possibilities of further developments like: the extension of the theory presented with predicates and quantifiers, a study of other aspects of control in proof-search, a development of this analysis for some substructural logics like fragments of linear logic, variants of bunched implications logic and finally a study of the relationships between semantics of reductive logic and proof-search proposed in this monograph and Girard's Ludics, starting from recent results about game semantics.

This research monograph should be of interest to readers of this Bulletin. The approach and the results are clearly and rigorously presented with comments and formal proofs to support them. The prerequisites are appropriate to the intended audience of research, i.e., logicians and theoretical computer scientists who can find, chapter by chapter, new results about these new semantic perspectives. With this research monograph the authors provide to researchers interested in foundations of proof-search some key points that allow to have a better understanding about the necessity and the impact of semantics for new formulations of logical systems in the proof-search perspective.

Didier Galmiche

LORIA — Université Henri Poincaré, Campus Scientifique, B.P. 239, 54506 Vandœuvre-les-Nancy Cedex, France. Didier.Galmiche@loria.fr.

Michael Mitzenmacher and Eli Upfal. ***Probability and computing: Randomized algorithms and probabilistic analysis.*** Cambridge University Press, Cambridge, 2005, 386 pp.

Over the past three decades, a significant amount of research in algorithms and complexity has been devoted to the study of randomness and computation. One of the two main themes in this area of research is the design and analysis of randomized algorithms, which have access to a stream of random bits for use during computation. Experience has shown that for many computational problems, it is easier to design an efficient randomized algorithm which has good performance bounds ("with high probability"), than to find an efficient deterministic algorithm with comparable bounds. One example of this phenomenon is primality testing. Although it is now known that a deterministic algorithm for primality testing does exist, for decades the only known algorithm was a very simple randomized algorithm. It is often the case that a randomized algorithm is very simple to describe, and the main challenge lies in analysing the algorithm and in proving that desirable performance guarantees hold with high probability. The second main theme in randomness and computation is the analysis of the expected performance (perhaps the quality of the output, or the running time) of a deterministic algorithm, when the inputs are random and chosen according to a probability distribution. This is called probabilistic analysis, and it is valuable because worst-case analysis of an algorithm does not capture the typical performance of the algorithm. Many of the mathematical techniques used in probabilistic analysis of algorithms are the same techniques used in the analysis of randomized algorithms. Therefore it makes sense for students to study these topics as a whole. This is the idea behind the ***Probability and Computing*** book.

Mitzenmacher and Upfal present the basic techniques for reasoning about probability and algorithms in an accessible way. The book is intended for advanced undergraduate and beginning graduate students.

There are fourteen chapters in ***Probability and Computing***. The early chapters mostly focus on basic probabilistic techniques. In the later chapters, the authors draw on their own areas of expertise to give a more comprehensive treatment of research topics such as Ball-in-Bins, the Markov chain Monte Carlo (MCMC) method, and Coding theory. However, the emphasis in this text is on techniques rather than applications.

Chapter 1 is an introduction to probability in the context of computer science. Definitions of concepts such as random event, the inclusion/exclusion principle, conditional probabilities, and Bayes' Law are given. These basic definitions are illustrated by examples of randomized algorithms for testing polynomial identities and verifying matrix multiplication. The authors also present and analyse the simple edge contraction algorithm for the Min-cut problem, proving that it succeeds with probability $2/n(n-1)$.

Chapters 2–4 present the techniques most commonly used by researchers in randomized algorithms. Chapter 2 (Discrete Random Variables and their Expectation) presents, and proves, basic facts about expectation such as linearity of expectation, Jansen's Inequality and conditional expectation, and also defines the Bernoulli and Binomial random variables. These ideas are then applied to the analysis of the coupon collector problem, and to the analysis of randomized Quicksort (and to the average-case analysis of deterministic Quicksort). Chapter 3 (Moments and Deviations) presents simple techniques for bounding the tail distribution of a random variable. It defines variance and moments, and proves Markov's Inequality and Chebyshev's Inequality. It then presents the randomized median-finding algorithm, and uses Chebyshev's Inequality to prove that the algorithm returns the median element with high probability. Chapter 4 (Chernoff Bounds) introduces Chernoff Bounds, which provide exponentially decreasing bounds on the tail distribution. The authors prove the Chernoff Bounds for the sum of a set of independent Poisson trials (the most commonly studied case of Chernoff Bounds). They use this result to analyse a randomized Set Balancing algorithm, and algorithms for permutation routing on the hypercube and on the butterfly network.

Chapter 5 (Balls, Bins, and Random Graphs) is more application-oriented than the earlier chapters. It introduces the Balls-in-Bins model and derives simple bounds on the probability that any bin receives a certain number of balls. It introduces Poisson random variables, and shows that the Poisson random variable can closely approximate the distribution of an individual bin in the Balls-in-Bins model. Applications of Balls-in-Bins to hashing are discussed. The second theme of Chapter 5 is the Random Graph model. The basic models $G_{n,p}$ and $G_{n,N}$ are introduced. The authors then present a randomized algorithm to find a Hamilton cycle (with high probability) in a random graph from $G_{n,p}$ when $p \geq 40 \ln n/n$.

Chapter 6 discusses the Probabilistic Method, where the goal is to prove existence of an object with certain desirable properties, by proving that these properties hold with positive probability when an object is randomly chosen from a particular set. In this chapter, the authors first present a randomized algorithm to find a large cut in a graph. Then they present standard results which are used for (probabilistic) proofs of existence, such as the Second Moment Method and the Lovasz Local Lemma. They also show how the probabilistic method can be made constructive in some cases. Examples presented in the chapter include threshold behaviours in the Random Graph model, proof of existence of edge-disjoint paths in certain graphs, and existence of a satisfying assignment for certain cases of k-SAT.

Chapter 7 is concerned with Markov chains and Random Walks, a theme which is continued in Chapters 10 and 11. This chapter presents the basic definitions for Markov chains. The authors prove that any finite ergodic Markov chain has a stationary distribution, a key result in probability theory. They define a Markov model of a simple queue and evaluate its stationary distribution. They consider random walks on undirected graphs, and discuss the cover time of such a walk. The chapter ends with a nice treatment of Parrando's paradox.

Chapter 8 (Continuous distributions and the Poisson process) is the only chapter of the book that concentrates on continuous distributions. The chapter introduces distributions such as the exponential distribution, the Poisson process (used to model arrival to a queue) and the continuous-time Markov process, which are important in computer science. The main example presented in this chapter is the Markovian queue, in particular, the $M/M/1$, $M/M/1/K$ and the $M/M/\infty$ queues in equilibrium. The chapter also contains a proof of the PASTA principle (Poisson arrivals see time averages) for the special case of $M/M/1$ in equilibrium.

Chapter 9 (Entropy, Randomness and Information) is concerned with entropy, coding theory and compression. The authors bound the average number of bits output by the best extraction function for a random variable X with entropy $H(X)$, for the special cases of a uniform random variable and a Poisson trial with parameter p. They then prove related results for compressing the outcome of a sequence of independent Poisson trials with parameter p. Finally, they prove Shannon's theorem for coding and decoding over a binary symmetric channel.

Chapters 10 and 11* are mostly concerned with random sampling. Chapter 10 (The Monte Carlo Method) begins by considering the basic Monte Carlo method. The authors first show how to use Chernoff bounds to bound the number of samples required to obtain an $(1 \pm \varepsilon)$ approximation of a ratio. The concept of an FPRAS (fully-polynomial randomized approximation scheme) is defined, and an FPRAS for approximately counting satisfying assignments for DNF formulae is presented. Next the concept of an FPAUS (fully polynomial almost-uniform sampler) is defined. The authors show how an FPAUS for sampling elements of a set can be converted to an FPRAS for approximating the cardinality of a set, by demonstrating how to perform this conversion for independent sets. The chapter concludes by defining the Markov chain Monte Carlo (MCMC) method, where sampling is carried out by defining a Markov chain with the appropriate stationary distribution, and running that chain for a large number of steps. Chapter 11* (Coupling of Markov chains) is a "starred" chapter, indicating more difficult content. This chapter is concerned with the mixing time (number of steps required to approach the stationary distribution) of a Markov chain. The authors introduce the Coupling technique for bounding mixing time. They state and prove the standard coupling lemma, and apply this lemma to prove that a natural card-shuffling chain mixes after $n \ln(n/\varepsilon)$ steps, along with other applications. They present a well-known bound (using coupling) on the mixing time of a Markov chain on proper colourings of a given graph, when the number of colours available is greater than twice the degree of the graph. They finish the chapter by informally introducing a simpler variant of coupling called path coupling, which is often sufficient for bounding mixing time. Part of the proof of the path coupling theorem is set as Exercise 11.7.

Chapter 12 is concerned with Martingales, which are sequences of random variables which arise in the analysis of random walks and in gambling problems. The concept of stopping time is defined, and the martingale stopping theorem, which proves that under certain conditions, expected return at the stopping time is equal to the value of the initial random variable, is stated without proof. Wald's equation, which is often used in the analysis of Las Vegas algorithms, is stated and proved. Finally the authors prove some tail inequalities for martingales, known as the Azuma-Hoeffding Inequalities, and apply these inequalities to problems in pattern matching and balls-in-bins.

Chapter 13 (Pairwise Independence and Universal Hash Functions) is concerned with reducing the amount of randomness required for certain applications. The authors define pairwise independence, and show how to construct $2^b - 1$ pairwise independent bits from b random bits. They use this to derandomize the large-cut algorithm of Chapter 6. They introduce Chebyshev's Inequality for pairwise independent values. They consider families of universal hash functions, and give examples of a 2-universal family of hash functions and a strongly 2-universal family of hash functions. Finally, they show how a family of 2-universal hash functions can be used to design a randomized algorithm to find heavy hitters in data streams.

Chapter 14* is a "starred" chapter on the subject of Balanced Allocations. The main theme of this chapter is how load balancing in the Balls-in-Bins model can be improved when a ball has "two choices" to choose from for its target bin. The authors apply Chernoff bounds in a long proof to show that the maximum load of a bin is significantly smaller in the "d choices" model than in the standard Balls-in-Bins model of Chapter 5, where each ball choses one random bin and is allocated to that bin. The authors then discuss applications of the power of two choices to hashing and dynamic resource allocation.

I enjoyed reading ***Probability and Computing*** and I would use it as a textbook if I were teaching a course on this subject (probably covering a subset of the topics). The intended audience is advanced undergraduates and beginning graduate students, and the material is presented at the right level for those students. I think that an introductory course on algorithms and an introductory course on complexity would both be prerequisites for the book, especially because computational complexity is mentioned in passing on a couple of occasions through the text. The book assumes less knowledge of probability, but it would be helpful if the reader had already taken a undergraduate probability course. The book is very well-written, in a student-friendly style, with many basic steps of proofs being explained in more detail than would be given in papers. Almost all the techniques and theorems which are presented in this book come with complete proofs. In each chapter, any techniques which are introduced in that chapter are tested out on a set of on example problems. The coupon collector problem and variants of satisfiability both recur as examples in different chapters of the text. Each chapter ends with a long set of exercises. The early exercises in each chapter ask the student to perform basic manipulations, but each chapter also poses some problems which require more creative thinking (harder questions are not highlighted on an individual basis, though the two 'starred' chapters have more difficult exercises). There are a few exploratory exercises throughout the book, which ask the student to write code and run experiments to test hypotheses.

In comparison to other books, the subject matter of ***Probability and Computing*** is most similar to the content of ***Randomized Algorithms***, by Motwani and Raghavan. However, the emphasis of the Motwani and Raghavan textbook tends more to particular applications of randomness (in areas such as Data Structures, Geometric Algorithms, Graph Algorithms, Approximate Counting, Parallel and Distributed Algorithms, Online Algorithms and Cryptography) than this text, which concentrates on giving a thorough presentation of the many techniques used in probabilistic analysis. ***Probability and Computing*** does contain some chapters which are devoted to particular applications, but the applications which receive most attention are Coding Theory, the Markov chain Monte Carlo method, and the Balls-in-Bins model.

The only small complaint I have about ***Probability and Computing*** is that no references are given in the book, apart from a list of about 15 more advanced probability textbooks for further reading. Most textbooks in algorithms and complexity give references to the papers where the material was originally published (usually by including a "notes" section at the end of each chapter). Mitzenmacher and Upfal give no references to any papers in this text. This is disappointing because some of the later chapters, especially the starred chapters on Coupling and on Balanced Allocations, would otherwise be an excellent jumping-off point for a PhD student interested in doing in-depth reading on a given topic. The lack of referencing also means that the authors do not "tell the story" of how particular areas of research developed over time. I think some extra background (and perhaps a bit more discussion of the complexity status of the problems in this text) would have improved the book. Despite this, I think this is an excellent text which will be useful for anyone studying randomized algorithms and probabilistic analysis.

MARY CRYAN

School of Informatics, University of Edinburgh, Edinburgh EH9 3JZ, Scotland, UK. mcryan@inf.ed.ac.uk.

ERIC SCHECHTER. ***Classical and nonclassical logic: an introduction to the mathematics of propositions.*** Princeton University Press, Princeton and Oxford, 2005, x + 507 pp.

This book is derived from and is designed for an introductory course in propositional logic that includes some nonclassical logics. It covers classical propositional logic, intuitionistic logic, Łukasiewicz's 3-valued and infinite-valued logics, several relevance logics and the implicational fragment of the modal logic S5.

There are six parts: Preliminaries, Semantics, Basic syntactics, One-formula extensions, Soundness and major logics, Advanced results. The first part occupies nearly half of the book. It is a long rambling discussion of elementary aspects of mathematics, philosophy, set theory, topology, and formal languages. It is intended only as background reading and could be skipped. Chapter 1 is an introduction for teachers. In Chapter 2, the introduction for students, the author says, "Regardless of previous background, however, any liberal arts undergraduate student might wish to take at least one course in logic, for the subject has great philosophical significance: It tell us something about our place in the universe. It tells us how we think—or more precisely, how we think we think. Mathematics contains eternal truths about number and shape; mathematical logic contains truths about the nature of truth itself." (pp. 20–21) " ... , but the abstract concept of number 3 itself does not exist as a physical entity anywhere in our real world. It exists only in our minds." (p. 27) This mix of the nineteenth century view of logic as psychology with a Platonist view of logic as about eternal truths may well leave students confused. Chapters 3 and 4 introduce set theory and topology (at the level of, for example, Venn diagrams). Chapter 5, English and informal classical logic, " ... is not intended to be mathematically precise." (p. 146) Here his discussion of language and bias is not well considered: "When mathematicians talk, they are attempting to copy ideas from the inside of one mathematician's head to the inside of another mathematician's head, but there is no way to be absolutely certain that the two heads end up containing identical mathematics." (p. 148) Much of this and the previous discussion uses the formal symbols from propositional logic, but the explanation of what they are meant to formalize comes only much later. Chapter 6, Definition of a formal language, has many good illustrations of use-mention difficulties: " ... "$\vee$" is not disjunction. It is the *disjunction symbol*, and it will be used to represent different kinds of disjunctions," (p. 217), so " ... $\vee$ and $\wedge$ are *not* commutative in some peculiar logics ... " (p. 217), but $\vee$ also denotes itself, "For instance, $\rightarrow\pi_3\pi_{11}\wedge\vee\neg\wedge$ is a string of length 6." (p. 209) The author thus seems to believe that a symbol may or may not be commutative.

The second half of the book (parts B to F) contains the presentation of the formal systems. All the semantical interpretations of the formal language for the various logical systems are presented in part B. Here the author mixes the formal language and the functional interpretation of the formal language: "Hereafter we will replace the uninterpreted symbols ... with the symbols ... which represent a particular interpretation." (p. 245–246) Axiom sets are presented, each built up in part D from the "basic" logic of part C by adding one or more formulas. He sometimes connects axiomatizations to semantics with a completeness theorem, but more often shows only that one or more of the semantic interpretations he has previously introduced are sound. Because of this arrangement, the first chapter of each part begins with a variation on this distressing advice: "This chapter should be read simultaneously with the next one or more chapters—i.e., the reader will have to flip pages back and forth. Apparently this arrangement is unavoidable. The present chapter contains several very abstract definitions; the next few chapters consist of many examples of those definitions". (p. 313) Although there is quite a variety of interesting technical material in this book, the presentations are often unclear and the terminology is peculiar. For example, intuitionistic logic is called "constructive logic", Łukasiewicz's countably-valued logic is called "Zadeh logic", it and Łukasiewicz's 3-valued logic are called fuzzy logics, the Deduction Theorem is called the "Deduction Principle", and so on.

Roger D. Maddux

Department of Mathematics, 396 Carver Hall, Iowa State University, Ames, IA 50011, USA. maddux@iastate.edu.