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Caicedo, Andrés Eduardo (1-MR);

Larson, Paul [Larson, Paul B.] (1-MMOH-DM); Sargsyan, Grigor (1-RTG);

Schindler, Ralf [Schindler, Ralf-Dieter] (D-MUNS-ML);

Steel, John [Steel, John Robert] (1-CA); Zeman, Martin (1-CA3)

Square principles in \mathbb{P}_{\max} extensions. (English summary)

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In this paper, the authors obtain a number of interesting combinatorial results by forcing over strong models of determinacy. For instance they obtain

$$\neg \square(\omega_2) + \neg \square_{\omega_2} + 2^{\aleph_1} = \aleph_2$$

from a determinacy assumption which is weaker than the existence of a Woodin cardinal which is a limit of Woodin cardinals. This consistency proof has a much weaker hypothesis and involves a completely novel approach compared with previous consistency proofs of this statement. Their approach involves forcing with \mathbb{P}_{\max} over a strong model of determinacy and then further forcing to well-order the power set of the reals. By using the analysis of the \mathbb{P}_{\max} forcing and HOD in these different models, the combinatorial results are obtained. In the paper, familiarity with the theory of AD^+ , fine structure, and \mathbb{P}_{\max} is assumed, although a number of important AD^+ results are reviewed. In addition, there are, generally speaking, copious references. *Scott Stefan Cramer*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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